Problem Set 7
Due Wednesday, May 23.

Real Analysis

Math 131A, Spring Quarter 2012

1. Do problems 15.2, 15.4, 17.5, 17.6, 17.10, 17.12, 17.13, 17.14, 18.4, in the textbook.
2. Let $f\colon [0,1] \to \mathbb{R}$ be a continuous function with $f(0) = f(1)$. Show that there is some $x \in [0,1]$ such that $f(x) = f(x + \frac{1}{2})$.
3. Let $S \subseteq \mathbb{R}$ and $x_0 \in S$. One says that x_0 is *isolated* if there is an $\varepsilon > 0$ such that $(x_0 - \varepsilon, x_0 + \varepsilon) \cap S = \{x_0\}$. Show that every function $f\colon S \to \mathbb{R}$ is continuous at each isolated $x_0 \in S$.
4. Let $g\colon S \to \mathbb{R}$ be continuous at $x_0 \in S$, and suppose $g(x_0) \neq 0$.
   (a) Show that there is an open interval I such that $x_0 \in I$ and $g(x) \neq 0$ for each $x \in I \cap S$. (Distinguish between the case where x_0 is isolated and non-isolated.)
   (b) Show that $1/g\colon I \cap S \to \mathbb{R}$ is continuous at x_0.