Problem Set 1
Due Friday, October 4.

*Real Analysis*

Math 131A, Fall Quarter 2013

1. Do problems 1.1, 1.5, 1.8, 1.9, 1.11, 2.4, 3.5, 3.8 in the textbook.
2. Prove that $(1+x)^n \geq 1+nx$ if $x \in \mathbb{R}$ with $1+x>0$, for all natural numbers $n \geq 1$. (Bernoulli's Inequality.)
3. Here is a proposed proof for the statement "all cats have the same color". We proceed as follows: By induction on $n$ we show that in any set of $n$ cats, there are no two cats with a different color. *Base step:* A set consisting of one cat only clearly satisfies the claim. *Inductive step:* Suppose the theorem has been proven for all sets of $n$ cats. Consider a set of $n+1$ cats. Take one cat out of the set; call it "first cat". By the inductive hypothesis, the rest of the cats (consisting of $n$ cats), are of the same color, say color $x$. Take another cat away; call it "second cat". Remember that the rest of the set has $n-1$ cats of color $x$. Now return the first cat to the set. The set has now $n$ cats, $n-1$ of which are color $x$; the first cat has some unknown color. Since by inductive hypothesis a set of cats with $n$ elements has just one color, the first cat must also have color $x$. Now bring the second cat back in, and we get back our set of $n+1$ cats, all of which have color $x$. To summarize, we assumed any $n$ cats were of color $x$, and we have proven that any $n+1$ cats are of color $x$ also. *Conclusion:* By virtue of mathematical induction, all cats in the world are of the same color. Since this conclusion is obviously false, only three possibilities are available: the principle of mathematical induction is false or inapplicable to cats, logic is false or inapplicable to cats, or there is an error in the proof. Which one? Explain why!