Problem Set 2
Due Friday, October 11.

Real Analysis

Math 131A, Fall Quarter 2013

1. Let F be a field, i.e., a set equipped with two maps

$$(a,b) \mapsto a+b\colon F \times F \to F, \quad (a,b) \mapsto a \cdot b\colon K \times K \to K$$

   satisfying the axioms (A1)–(A4), (M1)–(M4) and (DL) stated in class.

   (a) Show that the element 0 postulated to exist in (A3) is unique.

   (b) Show that the element 1 postulated to exist in (M3) is unique.

2. Do problems 3.3 and 3.4 in the textbook.
3. Do problems 4.1–4.4 in the textbook for (a), (b), (k), (u), (v).
4. Do problem 4.14 in the textbook.
5. Recall that the complex numbers $\mathbb{C}$ is the set of all numbers of the form $a+bi$ where $a, b \in \mathbb{R}$ and i is a number satisfying $i^2 = -1$. We add and multiply complex numbers in the natural way:

$$(a+bi) + (a'+b'i) = (a+a') + (b+b')i$$
$$(a+bi) \cdot (a'+b'i) = (aa'-bb') + (ab'+ba')i$$

   (a) Verify that using these operations, $\mathbb{C}$ becomes a field.

   (b) Show that there does *not* exist a binary relation $\leq$ on $\mathbb{C}$ so that $\mathbb{C}$ becomes an ordered field.