Problem Set 5

Due Friday, November 1.

Real Analysis

Math 131A, Fall Quarter 2013

1. Do problems 10.7, 10.10, 11.6, 11.8, 11.10 in the textbook.

2. Let $a > 0$ and $x_0 > 0$ be real numbers. Define

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right)$$

   for $n = 0, 1, 2, \ldots$.

   (a) Show that $(x_n)_{n\geq 1}$ is decreasing and $x_n \geq \sqrt{a}$ for $n = 1, 2, 3, \ldots$.

   (b) Why does this imply that $(x_n)_{n\geq 1}$ has a limit?

   (c) Let $x = \lim_{n\to\infty} x_n$. Show that $x = \sqrt{a}$.

3. Let (a_n) and (b_n) be bounded sequences of real numbers. Show that

   $$\limsup a_n + \liminf b_n \leq \limsup(a_n + b_n) \leq \limsup a_n + \limsup b_n.$$

   Give an example of a *single* pair of sequences (a_n), (b_n) for which both inequalities are strict.