Problem Set 6
Due Friday, May 15.

Real Analysis

Math 131A, Spring Quarter 2015

1. Do problems 17.5, 17.6, 17.10, 17.12, 17.13, 17.14 in the textbook.

2. Let $S \subseteq \mathbb{R}$. One says that an element x_0 of S is *isolated* if there is an $\varepsilon > 0$ such that $(x_0 - \varepsilon, x_0 + \varepsilon) \cap S = \{x_0\}$. Show that every function $f \colon S \to \mathbb{R}$ is continuous at each isolated $x_0 \in S$.

3. Let $g \colon S \to \mathbb{R}$ be continuous at $x_0 \in S$, and suppose $g(x_0) \neq 0$.

   (a) Show that there is an open interval I such that $x_0 \in I$ and $g(x) \neq 0$ for each $x \in I \cap S$. (Distinguish between the case where x_0 is isolated and non-isolated.)

   (b) Show that $1/g \colon I \cap S \to \mathbb{R}$ is continuous at x_0.