Course Announcement

*O-Minimality and Diophantine Geometry*

Math 223M, Winter Quarter 2011

MWF 1–1:50PM, Mathematical Sciences Building 5127

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**Description.** O-minimality is a property of ordered structures which yields results generalizing the classical finiteness theorems long known to hold for semialgebraic and subanalytic sets, such as the existence of cell decompositions and Whitney stratifications. This leads to the development of a kind of "tame topology" (already envisaged by Grothendieck). Although originating in model theory, the notion of an o-minimal structure has proven to be increasingly useful in the fields of real algebraic and analytic geometry. The general theory has even had applications to subjects as varied as Lie theory, economics, and neural networks.

In the last few years, somewhat surprising connections between o-minimality and diophantine geometry have emerged. The starting point of this was a theorem of Pila and Wilkie (2004) concerning the distribution of rational points on subsets of $\mathbb{R}^n$ which are definable in o-minimal structures. Combining this theorem with number-theoretic considerations and work of Peterzil and Starchenko on complex analysis in an o-minimal context, Pila and Zannier (2008) gave a new proof for the Manin-Mumford Conjecture (Raynaud's Theorem) on torsion points in complex abelian varieties. Most recently, Pila (2009) used similar ideas to prove certain open cases of another conjecture in diophantine geometry (the André-Oort Conjecture).

This course will be split into two parts. In the first half, we will introduce the o-minimality axiom and develop its main consequences. We will discuss basic facts about o-minimal structures, and give important examples of such structures. The second half of the course will be an introduction to the circle of ideas alluded to above, covering at least the Pila-Wilkie theorem.

**Prerequisites.** Some basic knowledge of first-order logic, model theory, and abstract algebra.

**Course text.** The following book will be a good companion for the first half of the course:

> Lou van den Dries, *Tame Topology and O-Minimal Structures,* London Math. Soc. Lecture Note Series, vol. 248, Cambridge University Press, Cambridge (1998).

Additional reading for the second half of the course:

T. Scanlon, *Counting special points: logic, diophantine geometry and transcendence theory,* AMS Current Events Bulletin 2011, `http://math.berkeley.edu/~scanlon/papers/csp.pdf`

K. Peterzil, S. Starchenko, *Tame complex analysis and o-minimality,* Proceedings of the ICM, Hyderabad, India, 2010, `http://math.haifa.ac.il/kobi/Icm-final.ps`

J. Pila, A. Wilkie, *The rational points of a definable set,* Duke Math. J. **133** (2006), no. 3, 591–616.

A. Wilkie, *Diophantine properties of sets definable in o-minimal structures,* J. Symbolic Logic **69** (2004), no. 3, 851–861.