Course Announcement

Construction of o-minimal structures

Math 223M, Fall Quarter 2018

MWF 10 am–10:50 am, MS 5148

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Office hours. by appointment; MS 5614.

Description. O-minimality is a property of ordered structures which yields results generalizing the classical finiteness theorems long known to hold for semialgebraic and subanalytic sets, such as the existence of cell decompositions and Whitney stratifications. This leads to a development of a kind of "tame topology" (envisaged by Grothendieck). Although originating in model theory, the notion of an o-minimal structure has proven to be useful in real algebraic and real analytic geometry, and the general theory has even had applications to subjects as varied as Lie theory, economics, and neural networks.

Most recently, o-minimality has also found surprising uses in diophantine geometry, among other things leading to (unconditional) proofs of important cases of the André-Oort Conjecture. An important role in these developments is played by an o-minimal structure denoted by $\mathbb{R}_{\text{an,exp}}$ (the ordered field of real numbers expanded by restricted analytic functions and the exponential function), because it defines all elementary functions (with suitable necessary restrictions on periodic ones such as sine and cosine).

The construction of o-minimal structures often uses ideas from elimination theory and resolution of singularities. In this course we will focus on such methods, using $\mathbb{R}_{\text{an,exp}}$ as our guiding example. We will introduce the o-minimality axiom and its main consequences, and then give a complete proof of the o-minimality of $\mathbb{R}_{\text{an,exp}}$. Time permitting, we will also discuss Wilkie's results on analytic continuation of germs of definable functions in $\mathbb{R}_{\text{an,exp}}$.

Prerequisites. Some basic knowledge of first-order logic, model theory, and abstract algebra should be sufficient. If in doubt about your background, ask me.

References.

J. Denef, L. van den Dries, *p-adic and real subanalytic sets,* Ann. of Math. (2) **128** (1988), no. 1, 79–138;

L. van den Dries, *Tame Topology and O-Minimal Structures,* London Math. Soc. Lecture Note Series, vol. 248, Cambridge University Press, Cambridge (1998).

L. van den Dries, A. Macintyre, D. Marker, *The elementary theory of restricted analytic fields with exponentiation,* Ann. of Math. (2) **140** (1994), no. 1, 183–205.

L. van den Dries, P. Speissegger, *The field of reals with multisummable series and the exponential function,* Proc. London Math. Soc. (3) **81** (2000), no. 3, 513–565.

J.-P. Rolin, P. Speissegger, A. Wilkie, *Quasianalytic Denjoy-Carleman classes and o-minimality,* J. Amer. Math. Soc. **16** (2003), no. 4, 751–777.

A. Wilkie, *Complex continuations of* $\mathbb{R}_{\text{an,exp}}$*-definable unary functions with a diophantine application,* J. Lond. Math. Soc. (2) **93** (2016), no. 3, 547–566.