Problem Set 4
Due Friday, November 10.

Foundations of Number Theory

Math 435, Fall 2006

1. (10+10 pts.)
   (a) Show that $n^2 + 3n + 5 \equiv (n-4)^2 \mod 11$, for every integer n.
   (b) Use (a) to show that there is no integer n such that $n^2 + 3n + 5$ is divisible by 121.
2. (20 pts.) Use induction on n to show that
$$2^{2n+1} \equiv 9n^2 - 3n + 2 \mod 54$$
for every $n \in \mathbb{N}$.
3. (10+5+10+5 pts.)
   (a) Show that $6|(10^i - 4^i)$ for every $i \in \mathbb{N}$. Use this to show that
$$10^{10^i - 4^i} \equiv 1 \mod 7.$$
(Hint: Euler-Fermat Theorem.)
   (b) Use (a) to show that $10^{10^i} \equiv 4^{4^i} \mod 7$ for every $i \in \mathbb{N}$.
   (c) Show that $4^{4^i} \equiv 4 \mod 7$ for every $i \in \mathbb{N}$.
   (d) Use (b) and (c) to determine the remainder of
$$\sum_{i=1}^{10} 10^{10^i}$$
upon division by 7.
4. (20 pts.) Find all primitive roots modulo 23 (up to congruence mod 23).
5. (10 pts.) Let m, n be positive integers with $m|n$. Show that $\varphi(m)|\varphi(n)$.
6. (20pts. extra credit.) Show that $2730|(n^{13} - n)$ for every $n \in \mathbb{N}$.