Agenda

- Introduction
- Application of Interest Rate Models
- Comparison of Models
- Validations
Introduction
Key financials EURm

<table>
<thead>
<tr>
<th></th>
<th>2014</th>
<th>2015</th>
<th>2016</th>
<th>2017</th>
<th>2018</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross written premiums</td>
<td>6,064</td>
<td>5,211(1)</td>
<td>5,048(1)</td>
<td>5,293</td>
<td>5,309</td>
</tr>
<tr>
<td>Earnings before taxes</td>
<td>378</td>
<td>398(1)</td>
<td>226(1)</td>
<td>265</td>
<td>295</td>
</tr>
<tr>
<td>Combined ratio (net) (P&amp;C)</td>
<td>99.6%</td>
<td>97.9%</td>
<td>98.1%</td>
<td>97.5%</td>
<td>96.8%</td>
</tr>
<tr>
<td>Operating Return on Equity</td>
<td>15,6%</td>
<td>17,2%</td>
<td>10,0%</td>
<td>10,2%</td>
<td>10,5%</td>
</tr>
</tbody>
</table>

(1) Excluding Italy

Diversification by regions and products (GWP 2018)

- **Ausgewogenes Portfolio**
  - 27% Lebensversicherung
  - 52% Schaden- und Unfallversicherung
  - 21% Krankenversicherung

- **... in den Kernmärkten Österreich und CEE**
  - 71% UNIQA Österreich
  - 29% UNIQA international
Typical value chain in the insurance business

Strategic starting points:
- Product design
- Product mix

1. **Product design**
   - Actuaries / Mathematicians are calculating the sufficient premiums and are key stakeholders.

2. **Finance**
   - Valuation of liabilities is one of the key tasks for actuaries / mathematicians, here you also have the link to financial mathematics
   - Asset management is another area where students with the background of financial mathematics are appreciated

3. **Risk Management**
   - Valuation models for the estimation of
     - expected profit and
     - underlying risk
     are developed from mathematicians
What is an interest rate curve?

<table>
<thead>
<tr>
<th>maturity</th>
<th>yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.33%</td>
</tr>
<tr>
<td>2</td>
<td>-0.28%</td>
</tr>
<tr>
<td>3</td>
<td>-0.18%</td>
</tr>
<tr>
<td>4</td>
<td>-0.05%</td>
</tr>
<tr>
<td>5</td>
<td>0.10%</td>
</tr>
<tr>
<td>6</td>
<td>0.24%</td>
</tr>
<tr>
<td>7</td>
<td>0.37%</td>
</tr>
<tr>
<td>8</td>
<td>0.50%</td>
</tr>
<tr>
<td>9</td>
<td>0.62%</td>
</tr>
<tr>
<td>10</td>
<td>0.73%</td>
</tr>
<tr>
<td>11</td>
<td>0.82%</td>
</tr>
<tr>
<td>12</td>
<td>0.91%</td>
</tr>
<tr>
<td>13</td>
<td>0.99%</td>
</tr>
<tr>
<td>14</td>
<td>1.05%</td>
</tr>
<tr>
<td>15</td>
<td>1.11%</td>
</tr>
<tr>
<td>16</td>
<td>1.14%</td>
</tr>
<tr>
<td>17</td>
<td>1.17%</td>
</tr>
<tr>
<td>18</td>
<td>1.20%</td>
</tr>
<tr>
<td>19</td>
<td>1.24%</td>
</tr>
<tr>
<td>20</td>
<td>1.28%</td>
</tr>
</tbody>
</table>
Where is an interest rate curve needed?

- Financial Mathematics:
  - Price of a bond:
    
    \[ P_T = e^{-\int_0^T r_t \, dt} \]
  
  - Price of a put option with the Black-Scholes formula (constant interest rate):
    
    \[ P(S_0, K, \sigma, r, T) = e^{-rT} K \Phi(-d_2) - S_0 \Phi(-d_1), \quad d_1 = \frac{\ln\left(\frac{S_0}{K}\right)+(r+\sigma^2/2)T}{\sigma \sqrt{T}}, \quad d_2 = d_1 - \sigma \sqrt{T} \]
  
  - i.e. discounting -> pricing

- Insurance:
  - Discounting / pricing
  - Forecasting / strategic planning
  - Risk capital
Important Note on „the“ Interest Rate Curve

The theory of financial markets needs a risk-free interest rate curve to ensure an arbitrage-free market.

Unfortunately a risk-free curve is not easy to define, as the recent crisis have demonstrated the lack of risk-free assets (and thus the instruments to derive the interest rate curve).

[Graph showing yield against maturity for different curves: Govie_DE and SII_curve.]

Alexander Filler
The balance sheet of an insurance company consists (simplified) of:

- **Assets** (e.g. bonds, equities, real estates, …)
- **Liabilities** (policies to customers, e.g. property insurance, motor insurance,…)

Where both sides of the balance sheet depend on interest rates:

- **Assets**: bonds – discounted future (fixed) cashflows
- **Liabilities**: (Expectation of) discounted cashflows
Question: How do assets (here bonds) and liabilities typically react on interest movements? In particular when the whole interest rate curve rises (e.g. 50bp)?
Answer: Both drop, typically liabilities more than bonds
History of Interest Rates

1-year-Swaprate
10-year-Swaprate
20-year-Swaprate

31.12.2019
30.09.2019
30.06.2019
31.03.2019
31.12.2018
30.09.2018
30.06.2018
31.03.2018
31.12.2017
30.09.2017
30.06.2017
31.03.2017
30.09.2016
30.06.2016
31.03.2016
31.12.2015
30.09.2015
30.06.2015
31.03.2015
Application Areas of Interest Rate Models at UNIQA
<table>
<thead>
<tr>
<th></th>
<th>Risk neutral scenarios</th>
<th>Inputs for valuation of liabilities with profit participation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Real world scenarios</td>
<td>Basis for the derivation of the Strategic Asset Allocation</td>
</tr>
<tr>
<td></td>
<td>forward looking</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Real world scenarios</td>
<td>Basis for the calculation of the risk capital requirement</td>
</tr>
<tr>
<td></td>
<td>historic</td>
<td></td>
</tr>
</tbody>
</table>
1. Risk neutral scenarios
   Inputs for valuation of liabilities with profit participation

2. Real world scenarios forward looking
   Basis for the derivation of the Strategic Asset Allocation

3. Real world scenarios historic
   Basis for the calculation of the risk capital requirement
Risk neutral scenarios are one of the main inputs for the valuation of liabilities with profit participation.

Description:

Policies with profit participation include an option for the customer. Typically the option is path dependent and the cashflows depend in a non-trivial way on simulated stochastic variables, e.g. interest rates or equity returns. Thus Monte Carlo simulations are used to determine the market values.
Risk Neutral Scenarios

Focus:

- Time horizon: 60 / 100 years
- Risk-free drift (i.e. martingale property for all simulated assets must be fulfilled)
- Market Implied Volatilities

Used / produced from (at UNIQA):

Risk managers produce simulations. Actuaries use simulations to calculate liabilities.
<table>
<thead>
<tr>
<th></th>
<th>Application Area</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Risk neutral scenarios</td>
<td>Inputs for valuation of liabilities with profit participation</td>
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<td>2</td>
<td>Real world scenarios forward looking</td>
<td>Basis for the derivation of the Strategic Asset Allocation</td>
</tr>
<tr>
<td>3</td>
<td>Real world scenarios historic</td>
<td>Basis for the calculation of the risk capital requirement</td>
</tr>
</tbody>
</table>
Real world scenarios - forward looking

Usage / Description:

Real word scenarios with drift are used to determine the strategic asset allocation (How many governmental bonds, equities, real estates,… should be in the portfolio in the next year(s)?)

Focus:
- Time horizon: 1-5 / 30 years
- expected drift and volatilities of the assets.

Used / produced from (at UNIQA):

Asset managers produce simulations and determine efficient portfolios.
<table>
<thead>
<tr>
<th></th>
<th>Application Areas at UNIQA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Risk neutral scenarios</td>
</tr>
<tr>
<td>2</td>
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</tr>
<tr>
<td></td>
<td>forward looking</td>
</tr>
<tr>
<td>3</td>
<td>Real world scenarios</td>
</tr>
<tr>
<td></td>
<td>historic</td>
</tr>
</tbody>
</table>
Real world scenarios - historic

Usage:

Real word scenarios without drift are used to determine the **solvency capital requirement** (i.e. the Value at Risk for a one-year-horizon of a 99.5% probability of „equity“).

Description:

To determine the capital requirement assets and liabilities are simulated (where the simulation of the interest rate curve is of central importance) to retrieve a distribution of „equity“. From this distribution the 99.5% quantile is the capital requirement.
Real world scenarios - historic

Focus:

- Time horizon: 1 year
- Tails of the empirical (historic) distribution

Used / produced from (at UNIQA):

Risk managers produce simulations and determine the risk capital.
Comparison of Models
Comparison of Models

Lots of models have been heavily discussed in literature in the last decades. „The“ model has not been found yet, thus, many models are used for all kind of usages.

<table>
<thead>
<tr>
<th>Application Area</th>
<th>Probability Measure</th>
<th>Model</th>
<th>Modelled variables / type of model</th>
<th>Focus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Valuation of liabilities</td>
<td>Risk neutral</td>
<td>Libor Market Model</td>
<td>(1-year) forward rates / structural model</td>
<td>time horizon: 100 years Drift: risk free Vols: Market implied volatilities</td>
</tr>
<tr>
<td>Strategic Asset Allocation</td>
<td>Real world – forward looking</td>
<td>Extended 2-factor Black-Karasinski Model</td>
<td>Short rate / structural model</td>
<td>time horizon: 30 years Drift: expected Vols: expected</td>
</tr>
<tr>
<td>Capital Requirement</td>
<td>Real world - historic</td>
<td>Principal Component Model</td>
<td>Spot rates / statistical model</td>
<td>time horizon: 1 year Drift: - Vols: empirical tails</td>
</tr>
</tbody>
</table>
## Comparison of Models

<table>
<thead>
<tr>
<th>Libor Market Model (2-Factor Forward Rate Model)</th>
<th>Black-Karasinski Model (2-Factor Short Rate Model)</th>
<th>Principal Component Model (3-Factor Spot Rate Model)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displaced LMM with stochastic volatility (&quot;LMM+&quot;)</td>
<td>Extended Black-Karasinski Model</td>
<td>Displaced lognormal Model with PCAs</td>
</tr>
<tr>
<td><strong>Forward rate model</strong> ($F_k$):</td>
<td><strong>Short rate model</strong> ($r$):</td>
<td><strong>Spot rate model</strong> ($r_l$):</td>
</tr>
<tr>
<td>[ \frac{dF_k(t)}{F_k(t)} = \frac{\delta}{\delta F_k(t)} \eta(t) \sum_{i=1}^{2} \sigma_i^2(t) dt + \sqrt{\eta(t)} \sum_{i=1}^{2} \sigma_i^2(t) dZ_i(t) ]</td>
<td>[ d \ln(r(t)) = \alpha_1 [\ln(m(t)) - \ln(r(t))] dt + \sigma_1 (dW^1_t + \gamma_1(t) dt) ]</td>
<td>[ r_l(t = 1) = (r_l(t = 0) + \delta^{IR}) e^{\sum_{i=1}^{3} \eta_{PC}(i) - \delta^{IR}} ]</td>
</tr>
<tr>
<td>$\eta(t)$ represents a stochastic volatility scaling factor:</td>
<td>$m(t)$ represents the mean-reversion level:</td>
<td>principal components $x_l$ are t-distributed</td>
</tr>
<tr>
<td>[ d \eta(t) = \alpha (\theta - \eta(t)) dt + \xi \sqrt{\eta(t)} dW_t ]</td>
<td>[ d \ln(m(t)) = \alpha_2 [\mu - \ln(m(t))] dt + \sigma_2 (dW^2_t + \gamma_2(t) dt) ]</td>
<td></td>
</tr>
</tbody>
</table>

![Risk Neutral 1 year horizon](image1)

![Real World - SSA 1 year horizon](image2)

![Real World - PBI 1 year horizon](image3)
Validations
Each of the three presented applications has a different focus on the model and its calibration due to their usages. Thus also the validation varies.

<table>
<thead>
<tr>
<th>Application Area</th>
<th>Probability Measure</th>
<th>Focus</th>
<th>Focus of Validations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Valuation of liabilities</td>
<td>Risk neutral</td>
<td>time horizon: 100 years</td>
<td>▪ Martingale property (risk free drift)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Drift: risk free</td>
<td>▪ Fit to market implied volatilities</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Vols: Market implied volatilities</td>
<td></td>
</tr>
<tr>
<td>Strategic Asset Allocation</td>
<td>Real world – forward looking</td>
<td>time horizon: 5 years</td>
<td>▪ Expected drifts and volatilities</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Drift: expected</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Vols: expected</td>
<td></td>
</tr>
<tr>
<td>Capital Requirement</td>
<td>Real world - historic</td>
<td>time horizon: 1 year</td>
<td>▪ Backtesting results</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Drift: -</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Vols: empirical tails</td>
<td></td>
</tr>
</tbody>
</table>
The central assumption of risk-neutral scenarios is that all assets in the expected value earn the risk-free interest rate (the 1-year forward rate every year).

So you test if the martingale property

\[ E(D_t S_t | F_0) = D_0 S_0 \]

wherby \( D_t \) the Deflator at the time \( t \), \( S_t \) the price index at the time \( t \) und \( F_0 \) the filtration at the time 0 is, fullfilled is with \( D_t = (1 + \hat{f}_{0,1})^{-1} \ldots (1 + \hat{f}_{t-1,t})^{-1} \), wherby \( \hat{f}_{t-1,t} \) the simulated 1-year forward rate at time \( t-1 \).

The martingale test is based on the Central Limit Theorem: When \( x_1, \ldots, x_N \) independently, identically distributed random numbers with expected value \( \mu \) and variance \( \sigma^2 \), the convergence in distribution

\[ \sqrt{N} \frac{\sum_{i=1}^{N} x_i - \mu}{\sigma} \xrightarrow{d} N(0,1) \]
For an index, the test is:

\[ \mu = E(D_t S_t | F_0) = 1 \]

\[
\sqrt{N} \frac{1}{N} \sum_{i=1}^{N} x_i - \mu \xrightarrow{N \to \infty} N(0, 1) \iff \frac{1}{N} \sum_{i=1}^{N} x_i \xrightarrow{N \to \infty} N(\mu, \sigma^2) / N
\]

And thus you get the confidence interval (with coverage probability of 95\%):

\[
\left[ \bar{x} - \frac{\sigma}{\sqrt{N}} \cdot z_{0.975}; \bar{x} + \frac{\sigma}{\sqrt{N}} \cdot z_{0.975} \right]
\]

With \( \bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i \), \( z_{0.975} \ldots 97.5\% \) quantile of the standard normal distribution. The test is thus passed if the expected value is within the confidence interval for each simulation time.
**Validation:**

Can you earn with a 10-year bond just an 1-year riskless forward rate?

\[ E[D_t I_t] = D_0 I_0 \]

\( I_t \) is the appropriate Total Return Index.
Validations
Risk Neutral Scenarios - Martingaltest

It could go dramatically wrong …
Validations
Real World Scenarios – historic - Backtesting

Backtesting t Dist (MLE) of Logreturns (annual shift)
of 3 PCs of SPOT_EUR_Level_EUR10 delta = 0.03
0 breaches, 0 %
Validations
Real World Scenarios – historic - Backtesting

Backtesting t Dist (MLE) of Logreturns (annual shift)
of 4 PCs of SPRD_EUR_GOHI_EUR GEN IT GOV 10Y delta = 0.05
9 breaches, 4.1 %

Backtesting t Dist (MLE) of Logreturns (annual shift)
of 4 PCs of SPRD_EUR_GOHI_EUR GEN IT GOV 10Y delta = 0.05
0 breaches, 0 %
Thanks for your attention!

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