Project Application: Lie Theory and Applications

Short description

Several lines of investigation will be persued, namely:

1. Invariant Theory. For example: If the coefficients of a polynomial depend smoothly on higher dimensional parameters in a manifold, when can we choose the roots in such a way that they also depend smoothly on these parameter?

2. Completely integrable systems and double Lie groups. For example, we want to study how to build new integrable systems from old ones by the use of Poisson mappings: notably multiplications on Poisson Lie groups.

3. Geodesics on infinite dimensional Lie groups and completely integrable systems. The geodesic equation on the Virasoro group of the right invariant H^0-metric is the Korteveg-De Vries equation. This allows for Jacobi fields, curvature etc. This should be extended to some other metrics and/or groups and lead to new infinite dimensional integrable systems.

4. Further investigations of infinite dimensional regular Lie groups. We want to answer the question, whether there exist non-regular convenient or even Fréchet-Lie-groups.

5. Attempts for a structure theory of the Lie algebra of vector fields on a finite dimensional Lie group. Our aim is to develop some representation theory for Lie algebras of vector fields on compact Lie groups and the associated infinite dimensional Lie groups. The aim is to develop first some 'algebraic structure theory' of vector fields.

6. Approximations procedures on regular Fréchet Lie groups aiming towards solving certain non-linear partial differential equations. We want to apply a new approximation procedure to nonlinear partial differential equations appearing naturally on Fréchet Lie groups, for example the KdV equation.

7. A non-linear version of Arzelà-Ascoli's theorem on convenient Lie groups. Can one detect compactness by Lipschitz metrics?

8. Actions of finite dimensional Lie groups and structures of orbit spaces. We want to obtain a better understanding of the geometry of the orbit space of an isometric Lie group action.

9. Actions of Lie algebras on manifolds. Following the investigation of the differential geometry of an action of a Lie algebra on a manifold, i.e. only an infinitesimal Lie group action, we want to study how this action can be extended to an enlarged manifold. Some results are already available.