MR0062694 (16,2l) 02.0X

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The unprovability of Suslin's hypothesis without the axiom of choice in the system of axioms of Bernays-Mostowski. (Russian)

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This paper has reference to an axiom-system $\mathfrak{S}$ for set-theory proposed by Mostowski [Fund. Math. **32**, 201–252 (1939), pp. 201–208]. This system $\mathfrak{S}$ is a modification of the system of Bernays [J. Symbolic Logic **2**, 65–77 (1937); **6**, 1–17 (1941); **7**, 65–89, 133–145 (1942); **8**, 89–106 (1943); **13**, 65–79 (1948); MR0003382 (2,210c); **3**, 290; **4**, 183; **5**, 198; **10**, 3]; it is similar to that of Gödel [Consistency of the continuum hypothesis, Princeton, 1940; MR0002514 (2,66c)]. The principal difference between Mostowski's system and those of Bernays and Gödel is that the former admits individuals (i.e. elements which are not sets); it also does not contain the axiom of choice, and was constructed for the specific purpose of showing the independence of the latter. The hypothesis of Suslin is the statement that if an ordered set is such that every non-overlapping set of open intervals is at most denumerable, then the ordered set has a denumerable subset. The thesis of this paper is to exhibit a model Z which satisfies the axiom of $\mathfrak{S}$ and contains an ordered subset (viz. the individuals) such that every set of non-overlapping intervals is denumerable but there is no dense denumerable subset. The paper describes the construction of Z and gives an outline of the proof that it has the properties stated. Putting in all the details of this proof would be a long and tedious process, and the reviewer has not attempted it; but it seems to be fairly straightforward. It also appears that the proof involves the axiom of choice at one point. The author points out that the axiom of choice is not valid for Z hence nothing is shown about independence if the axiom of choice is assumed.

Reviewed by *H. B. Curry*