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Esenin-Vol′pin, A. S.

An analysis of potential realizability. (Russian) 1959

Logičeskie issledovanija pp. 218–262 *Izdat. Akad. Nauk SSSR, Moscow*

In Markov's work on algorithm theory a distinction has been made between three levels of abstraction. On the first of these levels, called abstraction of equality, we ignore the distinctions between individual occurrences of words, and speak of words which are graphically identical as instances of the same (abstract) word. On the second level, where we have words generated by rules, we speak of words of any finite length (or processes of any finite number of steps) as attainable, even though there may not be room in the known universe, or time enough in the cosmic order, to actually write them down; this is known as the abstraction of potential actualizability (this word is better as translation of *osuščestvimost′* than 'realizability' because of conflicting technical uses of the later). On the third level, abstraction of actual infinity, we treat infinite sets and processes as actually existing. Intuitionists and other constructionists deny the third abstraction; extreme nominalists deny even the first; this paper is concerned with denial of the second. It is proposed to set up number theory and set theory which avoids commitment to this abstraction, and to do this in such a way that consistency can be established in the sense that no contradiction is actualizable. The paper claims to contain a sketch of a proof of such a program. It is admitted that complete rejection of abstraction would paralyze mathematics, and the author makes some compromising assumptions. In particular, certain relativizations of its notion of actualizability are discussed. The sketch as given is too rough for the reviewer to judge of the success of the author's program and the true significance of his results.

Reviewed by *H. B. Curry*