In accordance with the double title, this book has three purposes.

The first purpose is an exposition of Gaussian integral operators, i.e., operators having the form

$$Sf(x) = \int_{\mathbb{R}^n} \exp\Big\{ \frac{1}{2} \sum_{k,l} a_{kl} x_k x_l + \sum_{k,l} b_{kl} x_k y_l + \frac{1}{2} \sum_{k,l} c_{kl} y_k y_l \Big\} f(y)\, dy$$

Such operators appears in analysis, probability theory, and mathematical physics in numerous contexts; the most classical examples are the Fourier transform, the Poisson formula for solution of the heat equation, and the Mehler formula for evolution of the harmonic oscillator.

In fact, we treat 'Gaussian operators' in wider generality; for instance, we discuss Gaussian operators in the Fock spaces, the Segal–Bargmann transform, the Zak transform, operators with theta-kernels, Gaussian p-adic operators, the real-adelic correspondence etc.

The second purpose is to present a non-orthodox introduction to the classical groups. Basically, it is located in Chapters 2–3, which form an independent part of the book; also Chapter 10 contains a discussion of the p-adic case.

These two purposes do not coincide but they are not too diverse. Understanding of the Gaussian operators requires understanding of the classical groups. On the other hand, to make an exposition of the classical groups interesting and motivated it is useful to link them with something more vital; as representation theory, differential geometry, topology etc. A 'hybrid' of the classical groups with the Gaussian integral operators is one of several possible solutions.

The third purpose was to present an exposition of the 'Weil representation' (from historical point of view, it is the 'Friedrichs representation' or the 'Friedrichs–Segal–Berezin–Shale–Weil' representation, it is interesting that the first 4 authors of this chain had physical or math-physical motivations).

Representation theory of real semisimple Lie groups and noncommutative harmonic analysis were an important and dynamical branch of mathematics during 1940–70s. Now we have an obvious crisis of understandability and disappearance of an exterior interest to the subject. May be, this is not sad, since during the period mentioned above the theory had a deep influence on other branches of mathematics (and some of these branches are dynamical up to now, as the theory of multi-variant special functions, integrable systems, infinite dimensional groups).

However, it is reasonable to have 'democratic' books on this topic admissible for any mathematician or mathematical physicist. Also it seems that this (or not precisely this) theory admits points of a possible further growth. It is important to 'open' such points for exterior observation and to minimize a volume of 'self-education' that is necessary for coming to 'start positions'.

The topic and the style of this book were selected under the impress of the modern crisis. I had to reject an intention to write a monograph or a text-

book[1]. This text is a book on representation theory without development of any representation-theoretical machinery. I tried to follow a 'primitive-aggressive approach' (as [29]) and tried to avoid a necessity of self-learning of the reader in a process of reading; I also tried to minimize a dependence between different chapters.

The subject allows numerous possibilities for excursions in lateral directions and I am trying to use these opportunities. In such situations, I warn a reader that this is an 'excursion'. On the other hand, our subject admits an infinite growth from an arbitrary point. For this reason, I am very far from an idea to be complete at any place. For instance, the Gaussian operators in spaces of functions in infinite number of variables are more important in probability and mathematical physics than 'finite-dimensional' operators. Nevertheless they are not discussed at all. Also the Howe duality is not mentioned.

This book has obvious intersections with books of Lion, Vergne [96] and Folland [40]. I tried to avoid intersections with my own book [118], but Chapter 5 is an intersection. However I hope that all these intersections are not too large.

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[1]Several text-books on the representation theory were published relatively recently, see