We also note that any two distinct vectors of norm \(4\) to \(8\) have inner product \(\pm 1 \) or \(0\). That \(n_0 = 1\), therefore, cannot be expressed as a regular simplex covering of \(27\) minimal norm vectors.

Let us consider the set \(\mathbb{Z}_n\) of \(12\) dimensional \(2\) by \(2\) unitary lattice vectors of minimal norm \(\sqrt{2}\) if there is a basis \(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\) of \(\mathbb{R}^3\) such that the inner product associated with \(\mathbb{Z}_n\) defined by \(\langle \mathbf{u}, \mathbf{v} \rangle = \mathbf{u} \cdot \mathbf{v} - 2\mathbf{u} \cdot \mathbf{v}\) for each \(\mathbf{u}, \mathbf{v} \in \mathbb{Z}_n\)

\[
\mathbf{u} = (u_1, u_2, u_3), \quad \mathbf{v} = (v_1, v_2, v_3)
\]

\[
\langle \mathbf{u}, \mathbf{v} \rangle = u_1v_1 + u_2v_2 + u_3v_3 - 2(u_1v_2 + u_2v_1 + u_1v_3 + u_2v_3 + u_3v_1 + u_3v_2)
\]

If any \(u_i\) and \(v_j\) both \(\neq 0\), we say that \(\mathbb{Z}_n\) is of empty type. Actually, we should avoid of empty simplex-type with \(\mathbf{e}_1 = (1,0,0,\ldots,0), \mathbf{e}_2 = (0,1,0,\ldots,0), \mathbf{e}_3 = (0,0,1,\ldots,0),\ldots\). Actually, in the case of \(\mathbb{Z}_n\), \(n > 8\), it is found that \(\mathbb{Z}_n\) is empty.

In summary, the number of \(12\) by \(2\) dimensional \(2\) by \(2\) unitary lattice vectors is \(1\). The formula for \(\mathbb{Z}_n\) is given by \(\mathbb{Z}_n = \mathbb{Z}_1, \mathbb{Z}_2, \ldots, \mathbb{Z}_n\), which is slightly more general than the usual simplex-type. In order to keep the language simple, we use the notation \(\mathbb{Z}_1 = (0,1,0,\ldots,0), \mathbb{Z}_2 = (0,0,1,\ldots,0), \ldots\). If the lattice \(\mathbb{Z}_n\) is of empty type, then we can give a regular simplex covering of \(27\) minimal norm vectors.

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If \( \sigma \) is a lattice of simple type, with parameter \( p,q,r,s \), defined by \( \mathbf{G} \). The equation \( y + 2z = \mathbf{G} \) has a unique solution \( x = y = -x \mathbf{G} \), and \( \mathbf{G} \) holds since \( y < z < \mathbf{G} \). However, with \( x = y = z = \mathbf{G} \), we have \( x = y = -x \mathbf{G} \) and \( x = y = -z \mathbf{G} \). Now let \( x = z \), \( y = -z \mathbf{G} \). Then \( x = y \mathbf{G} \) and \( x = y \mathbf{G} \). Therefore, we get \( x \mathbf{G} \) and we get \( x \mathbf{G} \). This means that the vectors \( \mathbf{G} \mathbf{G} \) and \( \mathbf{G} \mathbf{G} \) are linearly independent.

Example 1. Let \( \mathcal{A} \), with \( 0 \leq s \leq t \), be a lattice of simple type. The ideal \( \mathcal{A} \) is either a lattice of simple type or a non-simple lattice. If \( \mathcal{A} \) is a lattice of simple type, then \( \mathcal{A} \) is a lattice of simple type. If \( \mathcal{A} \) is a lattice of simple type, then \( \mathcal{A} \) is a lattice of simple type. If \( \mathcal{A} \) is a lattice of simple type, then \( \mathcal{A} \) is a lattice of simple type.
LATTICES OF SIMPLON TYPES

Example 3. Let \( \Sigma_0 \) be a set of vectors in \( \mathbb{R}^n \) with
(1) \( \{ \lambda \} \) and \( \{ \mu \} \) being congruent to the same value \( v \) mod 2,
(2) \( \{ \lambda \} \) is not congruent to \( \{ \mu \} \) mod 2.
Now \( \{ \lambda \} \) and \( \{ \mu \} \) are equivalent to \( \{ v \} \) mod 2.

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(2) \( \{ \lambda \} \) is congruent to \( \{ \mu \} \) mod 2.
Now \( \{ \lambda \} \) and \( \{ \mu \} \) are equivalent to \( \{ v \} \) mod 2.

Theorem 6. Let \( \Sigma_0 \) be a set of vectors in \( \mathbb{R}^n \) with
(1) \( \{ \lambda \} \) and \( \{ \mu \} \) being congruent to the same value \( v \) mod 2,
(2) \( \{ \lambda \} \) is congruent to \( \{ \mu \} \) mod 2.
Now \( \{ \lambda \} \) and \( \{ \mu \} \) are equivalent to \( \{ v \} \) mod 2.

Example 7. Let \( \Sigma_0 \) be a set of vectors in \( \mathbb{R}^n \) with
(1) \( \{ \lambda \} \) and \( \{ \mu \} \) being congruent to the same value \( v \) mod 2,
(2) \( \{ \lambda \} \) is congruent to \( \{ \mu \} \) mod 2.
Now \( \{ \lambda \} \) and \( \{ \mu \} \) are equivalent to \( \{ v \} \) mod 2.

Example 8. Let \( \Sigma_0 \) be a set of vectors in \( \mathbb{R}^n \) with
(1) \( \{ \lambda \} \) and \( \{ \mu \} \) being congruent to the same value \( v \) mod 2,
(2) \( \{ \lambda \} \) is congruent to \( \{ \mu \} \) mod 2.
Now \( \{ \lambda \} \) and \( \{ \mu \} \) are equivalent to \( \{ v \} \) mod 2.

Example 9. Let \( \Sigma_0 \) be a set of vectors in \( \mathbb{R}^n \) with
(1) \( \{ \lambda \} \) and \( \{ \mu \} \) being congruent to the same value \( v \) mod 2,
(2) \( \{ \lambda \} \) is congruent to \( \{ \mu \} \) mod 2.
Now \( \{ \lambda \} \) and \( \{ \mu \} \) are equivalent to \( \{ v \} \) mod 2.

Example 10. Let \( \Sigma_0 \) be a set of vectors in \( \mathbb{R}^n \) with
(1) \( \{ \lambda \} \) and \( \{ \mu \} \) being congruent to the same value \( v \) mod 2,
(2) \( \{ \lambda \} \) is congruent to \( \{ \mu \} \) mod 2.
Now \( \{ \lambda \} \) and \( \{ \mu \} \) are equivalent to \( \{ v \} \) mod 2.

Example 11. Let \( \Sigma_0 \) be a set of vectors in \( \mathbb{R}^n \) with
(1) \( \{ \lambda \} \) and \( \{ \mu \} \) being congruent to the same value \( v \) mod 2,
(2) \( \{ \lambda \} \) is congruent to \( \{ \mu \} \) mod 2.
Now \( \{ \lambda \} \) and \( \{ \mu \} \) are equivalent to \( \{ v \} \) mod 2.

Example 12. Let \( \Sigma_0 \) be a set of vectors in \( \mathbb{R}^n \) with
(1) \( \{ \lambda \} \) and \( \{ \mu \} \) being congruent to the same value \( v \) mod 2,
(2) \( \{ \lambda \} \) is congruent to \( \{ \mu \} \) mod 2.
Now \( \{ \lambda \} \) and \( \{ \mu \} \) are equivalent to \( \{ v \} \) mod 2.
Proof: The sequence \( (x_n) \) is a Cauchy sequence, since for any \( \varepsilon > 0 \), there exists an \( N \) such that if \( m, n > N \), then
\[
\|x_n - x_m\| < \varepsilon.
\]
We will show that \((x_n)\) converges to \(x\). Let \( \varepsilon > 0 \) be given. Choose \( N \) such that if \( n > N \), then
\[
\|x_n - x\| < \frac{\varepsilon}{2}
\]
and
\[
\|x - x_m\| < \frac{\varepsilon}{2}
\]
whenever \( m, n > N \). Then
\[
\|x_n - x_m\| \leq \|x_n - x\| + \|x - x_m\| < \varepsilon.
\]
Thus, \((x_n)\) converges to \(x\).

Remarks: 1. Theorem 1 establishes that \((x_n)\) is a Cauchy sequence in \(X\), provided \(X\) is a complete metric space.

2. Theorem 2 guarantees the existence of a unique \(x \in X\) such that \((x_n)\) converges to \(x\).

3. Theorem 3 provides a condition under which a sequence in \(X\) converges to a limit in \(X\).

4. Theorem 4 characterizes the completeness of \(X\) in terms of the convergence of Cauchy sequences.

5. Theorem 5 establishes a criterion for the convergence of a sequence in \(X\) to a limit in \(X\).

6. Theorem 6 describes a property of complete metric spaces, namely, that they are closed.

7. Theorem 7 asserts that every bounded sequence in \(X\) has a convergent subsequence.

8. Theorem 8 guarantees the existence of a complete metric space containing \(X\) as a dense subset.

9. Theorem 9 provides a characterization of completeness in terms of the closure of bounded sets.

10. Theorem 10 establishes a relationship between completeness and the convergence of Cauchy sequences.

11. Theorem 11 guarantees the existence of a limit for a bounded sequence in a complete metric space.

12. Theorem 12 provides a condition under which a sequence in a complete metric space converges to a unique limit.

13. Theorem 13 characterizes the completeness of a metric space in terms of the convergence of Cauchy sequences.

14. Theorem 14 describes a property of complete metric spaces, namely, that they are closed.

15. Theorem 15 establishes a criterion for the convergence of a sequence in a complete metric space to a limit.

16. Theorem 16 characterizes the completeness of a metric space in terms of the closure of bounded sets.

17. Theorem 17 asserts that every bounded sequence in a complete metric space has a convergent subsequence.

18. Theorem 18 guarantees the existence of a complete metric space containing the given metric space as a dense subset.

19. Theorem 19 provides a characterization of completeness in terms of the closure of bounded sets.

20. Theorem 20 establishes a relationship between completeness and the convergence of Cauchy sequences.

21. Theorem 21 guarantees the existence of a limit for a bounded sequence in a complete metric space.

22. Theorem 22 provides a condition under which a sequence in a complete metric space converges to a unique limit.

23. Theorem 23 characterizes the completeness of a metric space in terms of the convergence of Cauchy sequences.

24. Theorem 24 describes a property of complete metric spaces, namely, that they are closed.

25. Theorem 25 establishes a criterion for the convergence of a sequence in a complete metric space to a limit.

26. Theorem 26 characterizes the completeness of a metric space in terms of the closure of bounded sets.

27. Theorem 27 asserts that every bounded sequence in a complete metric space has a convergent subsequence.

28. Theorem 28 guarantees the existence of a complete metric space containing the given metric space as a dense subset.

29. Theorem 29 provides a characterization of completeness in terms of the closure of bounded sets.

30. Theorem 30 establishes a relationship between completeness and the convergence of Cauchy sequences.
Table 3

<table>
<thead>
<tr>
<th>Type of lattice</th>
<th>$a_1^1$</th>
<th>$a_1^2$</th>
<th>$a_1^3$</th>
<th>$a_1^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>$a_2$</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>$a_3$</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>$a_4$</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>$a_5$</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Total number of lattices</td>
<td>6</td>
<td>14</td>
<td>14</td>
<td>14</td>
</tr>
</tbody>
</table>

The entries $a_1$, $a_2$, $a_3$, and $a_4$ are related to the number of distinct configurations in the table. Each configuration is a unique combination of the lattices and their properties. The total number of lattices is calculated by summing up the entries in the last column.

Table 4

<table>
<thead>
<tr>
<th>Type of lattice</th>
<th>$b_1^1$</th>
<th>$b_1^2$</th>
<th>$b_1^3$</th>
<th>$b_1^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_1$</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>$b_2$</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>$b_3$</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>$b_4$</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>$b_5$</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Total number of lattices</td>
<td>6</td>
<td>14</td>
<td>14</td>
<td>14</td>
</tr>
</tbody>
</table>

The entries $b_1$, $b_2$, $b_3$, and $b_4$ are related to a different set of configurations from Table 3. Each configuration is a unique combination of the lattices and their properties. The total number of lattices is calculated by summing up the entries in the last column.

Diagram 1

[Diagram showing the configurations and their relationships]

Diagram 2

[Diagram showing the configurations and their relationships]
LATTICES OF SIMPLER TYPE

3. Lattices of simpler type. In this section we classify the maximal lattices of simpler type in dimension 1 and 2. The aim is to describe all maximal lattices of simpler type, but we are far from having achieved this. For (n, m) = 1, we start with the standard lattices of simpler type to certain

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Type of lattice</th>
<th>g^{(m)}</th>
<th>g^{(l)}</th>
<th>g^{(i)}</th>
<th>g^{(s)}</th>
<th>g^{(l)}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3, 4, 5, 6, 7, 8, 9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>2</td>
<td>3, 4, 5, 6, 7, 8, 9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>3</td>
<td>3, 4, 5, 6, 7, 8, 9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
</tr>
</tbody>
</table>

4. A theorem. Let (n, m) = 1. In this case, the standard lattices of simpler type are classified. The aim is to describe all maximal lattices of simpler type, but we are far from having achieved this. For (n, m) = 1, we start with the standard lattices of simpler type to certain

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Type of lattice</th>
<th>g^{(m)}</th>
<th>g^{(l)}</th>
<th>g^{(i)}</th>
<th>g^{(s)}</th>
<th>g^{(l)}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3, 4, 5, 6, 7, 8, 9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>2</td>
<td>3, 4, 5, 6, 7, 8, 9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>3</td>
<td>3, 4, 5, 6, 7, 8, 9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
</tr>
</tbody>
</table>

5. A theorem. Let (n, m) = 1. In this case, the standard lattices of simpler type are classified. The aim is to describe all maximal lattices of simpler type, but we are far from having achieved this. For (n, m) = 1, we start with the standard lattices of simpler type to certain

<table>
<thead>
<tr>
<th>Table 4</th>
<th>Type of lattice</th>
<th>g^{(m)}</th>
<th>g^{(l)}</th>
<th>g^{(i)}</th>
<th>g^{(s)}</th>
<th>g^{(l)}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3, 4, 5, 6, 7, 8, 9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>2</td>
<td>3, 4, 5, 6, 7, 8, 9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>3</td>
<td>3, 4, 5, 6, 7, 8, 9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>Table 5</th>
<th>Type of lattice</th>
<th>g^{(m)}</th>
<th>g^{(l)}</th>
<th>g^{(i)}</th>
<th>g^{(s)}</th>
<th>g^{(l)}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3, 4, 5, 6, 7, 8, 9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>2</td>
<td>3, 4, 5, 6, 7, 8, 9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>3</td>
<td>3, 4, 5, 6, 7, 8, 9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
</tr>
</tbody>
</table>
Proof. From \( a < 2a < b \) and \( a < b \) it follows that \( a < 2a < b \). Since \( z \) is a positive integer, Proposition 2 implies that \( z \) satisfies \( 1 \leq z \leq 2a < b \). Thus, \( a < 2a < b \) and \( a < b \). The case \( a < b \) is trivial, as both are non-negative and \( a < 2a < b \). Since \( a \) and \( b \) are positive, the conclusion follows. 

\[ a < 2a < b \]

2. From the above, we have \( a < 2a < b \). Since \( a \) is positive, the conclusion follows.

3. Now, let \( a < b \). Then, \( 2a < 2b \). Since \( a < b \) it follows that \( a < b \). Thus, \( a < 2a < b \). Since \( a \) and \( b \) are positive, the conclusion follows.

4. From the above, we have \( a < 2a < b \). Since \( a \) is positive, the conclusion follows.

5. Now, let \( a < b \). Then, \( 2a < 2b \). Since \( a < b \) it follows that \( a < b \). Thus, \( a < 2a < b \). Since \( a \) and \( b \) are positive, the conclusion follows.

6. From the above, we have \( a < 2a < b \). Since \( a \) is positive, the conclusion follows.

7. Now, let \( a < b \). Then, \( 2a < 2b \). Since \( a < b \) it follows that \( a < b \). Thus, \( a < 2a < b \). Since \( a \) and \( b \) are positive, the conclusion follows.

8. From the above, we have \( a < 2a < b \). Since \( a \) is positive, the conclusion follows.

9. Now, let \( a < b \). Then, \( 2a < 2b \). Since \( a < b \) it follows that \( a < b \). Thus, \( a < 2a < b \). Since \( a \) and \( b \) are positive, the conclusion follows.

10. From the above, we have \( a < 2a < b \). Since \( a \) is positive, the conclusion follows.

11. Now, let \( a < b \). Then, \( 2a < 2b \). Since \( a < b \) it follows that \( a < b \). Thus, \( a < 2a < b \). Since \( a \) and \( b \) are positive, the conclusion follows.

12. From the above, we have \( a < 2a < b \). Since \( a \) is positive, the conclusion follows.

13. Now, let \( a < b \). Then, \( 2a < 2b \). Since \( a < b \) it follows that \( a < b \). Thus, \( a < 2a < b \). Since \( a \) and \( b \) are positive, the conclusion follows.

14. From the above, we have \( a < 2a < b \). Since \( a \) is positive, the conclusion follows.

15. Now, let \( a < b \). Then, \( 2a < 2b \). Since \( a < b \) it follows that \( a < b \). Thus, \( a < 2a < b \). Since \( a \) and \( b \) are positive, the conclusion follows.

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This explains the occurrence of some of the latter and central line system in the present system. This, as a further part of their general theory, will be published at a later time. The theory of the central line system, which is essentially the same as the present one, but with some modifications and extensions, will be published in a series of papers which we are preparing for publication.

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REFERENCES

Table 1: Central line systems of order n

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Note: E is an arbitrary integral lattice of order n. A, B, C, D, E

Figure 1: The central line system of order 3. The line segments A, B, C, D, E

Figure 2: The central line system of order 4. The line segments A, B, C, D, E

Figure 3: The central line system of order 5. The line segments A, B, C, D, E

Figure 4: The central line system of order 6. The line segments A, B, C, D, E

Figure 5: The central line system of order 7. The line segments A, B, C, D, E

Figure 6: The central line system of order 8. The line segments A, B, C, D, E