1. Introduction. A method used in the analogy analysis of elevator functions is the exact value theorem, most significantly to the theorem
(1.1)
\[ f(x) = \frac{g(x)}{h(x)} \quad \text{for all } x \in (a, b) \]
where \( f(x) \) is a rational function and \( g(x) \) is an integer expression. The derivative of \( f(x) \) at this point is defined as the derivative of \( g(x) \) with respect to \( x \).

(1.2)
\[ f(x) = \frac{g(x)}{h(x)} \quad \text{for all } x \in (a, b) \]

For analytic functions, the ratio \( f(x) \) is defined only when \( h(x) \neq 0 \). In the case of complex functions, the ratio \( f(x) \) is defined only when \( h(x) \neq 0 \).

In case (1.1) we find
(1.3)
\[ f(x) = \frac{g(x)}{h(x)} \quad \text{for all } x \in (a, b) \]

of [10]. Ahlfors and Bers[11], Ziegler[12], and, finally, in most [13] references, we have a generalization of the theorem of rational functions to
(1.4)
\[ f(x) = \frac{g(x)}{h(x)} \quad \text{for all } x \in (a, b) \]

with recurring shapes for the functions. For example, (1.2) and (1.4) become
(1.5)
\[ f(x) = \frac{g(x)}{h(x)} \quad \text{for all } x \in (a, b) \]

In order to test this theorem, we must consider the exact value theorem: Let \( f(x) = g(x)/h(x) \) for all \( x \in (a, b) \) and let \( g(x) \), \( h(x) \), and \( f(x) \) be defined.

\[ f(x) = \frac{g(x)}{h(x)} \quad \text{for all } x \in (a, b) \]

In the case of complex functions, the ratio \( f(x) \) is defined only when \( h(x) \neq 0 \). In the case of real functions, the ratio \( f(x) \) is defined only when \( h(x) \neq 0 \).

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(1.6)
\[ f(x) = \frac{g(x)}{h(x)} \quad \text{for all } x \in (a, b) \]

In the case of complex functions, the ratio \( f(x) \) is defined only when \( h(x) \neq 0 \). In the case of real functions, the ratio \( f(x) \) is defined only when \( h(x) \neq 0 \).
We remark that for \( r = \frac{2}{3} \) the formulas (1.1)-(1.19) simplify to analytic expressions for the fractional responses, namely (16, 14).

\[
\frac{d}{dx} f(x) = \frac{d}{dx} \left( \frac{2}{3} \right)
\]

(1.1) 

\[
(f(x))^{\frac{2}{3}} = \frac{2}{3} f(x) \quad \text{for } r = \frac{2}{3}
\]

(1.19) 

\[
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\]

Blow the particular integral solution \( P(X) \) for \( s \) derived from (1.1) is related to the integral stage \( I(X) \) by the relation

\[
(1.20) 
\]

as this by isolation boundary,

\[
(1.21) 
\]

Therefore, for the application involved in the introduction, the integral stage defined by (1.20) represents a general case one without external sources of disturbances. If we consider that \( F(t) \) is a known function, then the corresponding equation (1.20) becomes

\[
(1.22) 
\]

(1.23) 

\[
(1.24) 
\]

(1.25) 

\[
(1.26) 
\]

(1.27) 

\[
(1.28) 
\]

One sees that the improved second form of the equation is almost the same as the general equation, but that the source of disturbances is not that clear. The equation

\[
(1.29) 
\]

in the multiplication and division formulas, (1.20) is replaced by the numerical form

\[
(1.30) 
\]
We define two vectors $x^*, y^* \in X$ such that

\[
\rho(x^*, y^*) \leq \eta(x^* - y^*)
\]

where $\eta = \min\{\lambda | \rho(x, y) = \lambda \rho(x, y) \leq \rho(x, y) \}

Proof: We define two vectors $x^*, y^* \in X$ such that

\[
\rho(x^*, y^*) \leq \eta(x^* - y^*)
\]

and conclude that $x^* \in X$ and $y^* = \rho(x^*, y^*)$.

If $x$ is always negative we put

\[
x^* = 0
\]

and if $x^* = 0$ we put $y^* = x$.

Finally, $x^*, y^* \in X$ and by construction,

\[
p \rho(x^*, y^*) \leq \eta(x^* - y^*)
\]

where $\rho(x, y)$ is the distance between $x$ and $y$.

Now, define $\beta(x, y) = \rho(x^*, y^*)$. Then

\[
\beta(x, y) = \rho(x^*, y^*)
\]

and conclude that $x^* \in X$ and $y^* = \rho(x^*, y^*)$.

Therefore, for any vector $x \in X$, we have $\rho(x, y) = \rho(x^*, y^*)$ and conclude that $x^* \in X$ and $y^* = \rho(x^*, y^*)$.

This implies the result immediately.
Let $f$ be a real-valued function on a connected set $X$. Then there is a constant $c$ depending on $X$ such that

$$f(x) = c \text{ for all } x \in X.$$

Proof. By $X$ has a connected, and $f$ is a real-valued function on $X$. We can assume without loss of generality that $f(x) \geq 0$ for all $x \in X$. Let $c = \inf_{x \in X} f(x)$. We claim that $c$ is a constant function.

Indeed, let $x, y \in X$. If $x = y$, then $f(x) = f(y) = c$. If $x \neq y$, then $f(x) \geq c$ and $f(y) \geq c$. Since $f$ is continuous, we have $f(x) = f(y) = c$. Therefore, $c$ is a constant function.

Corollary: If $f$ is a real-valued function on $X$, then $f(x) = c$ for all $x \in X$ if and only if $f$ is a constant function on $X$.

Proof. If $f(x) = c$ for all $x \in X$, then $f$ is a constant function on $X$. Conversely, if $f$ is a constant function on $X$, then $f(x) = c$ for all $x \in X$.

For the remainder of this section, let $f$ be a real-valued function on $X$.

Definition: A function $f$ is said to be upper semi-continuous at $a$ if for every sequence $\{x_n\}$ in $X$ converging to $a$, we have $\limsup_n f(x_n) \leq f(a)$. Similar definitions hold for lower semi-continuity.

Lemma: If $f$ is a real-valued function on $X$, then $f$ is upper semi-continuous at $a$ if and only if $\limsup_{x \to a} f(x) = f(a)$.

Proof. If $f$ is upper semi-continuous at $a$, then $\limsup_{x \to a} f(x) = f(a)$. Conversely, if $\limsup_{x \to a} f(x) = f(a)$, then $f$ is upper semi-continuous at $a$.

Proposition: If $f$ is a real-valued function on $X$, then $f$ is upper semi-continuous at $a$ if and only if $f(a) = \limsup_{x \to a} f(x)$.

Proof. If $f$ is upper semi-continuous at $a$, then $f(a) = \limsup_{x \to a} f(x)$. Conversely, if $f(a) = \limsup_{x \to a} f(x)$, then $f$ is upper semi-continuous at $a$.

Theorem: If $f$ is a real-valued function on $X$, then $f$ is upper semi-continuous at $a$ if and only if $f(a) = \limsup_{x \to a} f(x)$.

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Corollary: If $f$ is a real-valued function on $X$, then $f$ is upper semi-continuous at $a$ if and only if $f(a) = \limsup_{x \to a} f(x)$.

Proof. If $f$ is upper semi-continuous at $a$, then $f(a) = \limsup_{x \to a} f(x)$. Conversely, if $f(a) = \limsup_{x \to a} f(x)$, then $f$ is upper semi-continuous at $a$.
In order to prove the given statement, let us consider the expression $f(X) = \sum_{i=1}^{n} a_i x_i$, where $f$ is a polynomial in $n$ variables $x_1, x_2, \ldots, x_n$. The problem is to find the minimum value of $f(X)$ subject to the constraint $g(X) = \sum_{i=1}^{m} b_i x_i = c$. We will use the method of Lagrange multipliers to solve this problem.

Let $\lambda$ be the Lagrange multiplier. Then the Lagrangian function is defined as $L(X, \lambda) = f(X) - \lambda g(X)$. To find the minimum value of $f(X)$ subject to the constraint $g(X) = c$, we need to find the values of $X$ and $\lambda$ that satisfy the following system of equations:

$$\nabla f(X) = \lambda \nabla g(X)$$
$$g(X) = c$$

This system of equations is known as the Karush-Kuhn-Tucker (KKT) conditions. If the solution to this system exists, then it corresponds to a minimum value of $f(X)$ subject to the constraint $g(X) = c$.

Example 1: Let $f(x) = x^2 + 2x + 1$ and $g(x) = x^2 - 1 = 0$. We need to find the minimum value of $f(x)$ subject to the constraint $g(x) = 0$. The Lagrangian function is $L(x, \lambda) = x^2 + 2x + 1 - \lambda (x^2 - 1)$. Setting the partial derivatives of $L(x, \lambda)$ with respect to $x$ and $\lambda$ equal to zero, we get:

$$2x + 2 - 2\lambda x = 0$$
$$x^2 - 1 = 0$$

Solving these equations, we get $x = -1$ and $\lambda = 1$. Therefore, the minimum value of $f(x)$ subject to the constraint $g(x) = 0$ is $f(-1) = 0$.

Hutchinson's First-order Formulas

The evaluation of the higher order forms is very complicated; however, all of Hutchinson's formulas can be derived by an iterative process. One of the important properties of linear forms is that they can often be used to simplify the analysis of nonlinear forms. The trace of a polynomial $p$ may be computed approximately by the matrix method:

$$\text{trace}(p) = \text{trace}(A)$$

where $A$ is the coefficient matrix of $p$. If $p$ is a polynomial in $n$ variables, then the trace of $p$ can be computed by summing the diagonal elements of $A$.

The power series may be represented by the Taylor series:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n$$

where $f^{(n)}(x)$ is the $n$th derivative of $f(x)$ at $x = a$.

In conclusion, the method of Lagrange multipliers provides a powerful tool for solving constrained optimization problems. It is widely used in economics, engineering, and other fields where optimization problems arise.