DUALITY IN COHERENT CONFIGURATIONS

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By introducing a duality concept in coherent algebra (i.e., adjacency algebra of coherent configurations) we give a new perspective to Delorme's duality theory for association schemes.

In particular we show that intersection matrices and positive semidefinite matrices, D-decompositions, eigenkeine graphs and spherical 2-designs, distance regular graphs and Delorme Burnside modules of dual schemes. Several 'almost dual' varieties which are not fully understood are also reported.

In his thesis, Delorme [1] observed a formal duality in the theory of association schemes which becomes an actual duality if the set is an abelian group. In this note we shall develop certain aspects of this duality in the slightly more general context of coherent algebras. In particular, we turn out that graphs and distance matrices, introduced in Neumaer [2], are dual objects.

A coherent algebra is an algebra \( A \) of square complex, \( n \times n \)-matrices indexed by a set \( X \) with a dynamical closed under compositionwise (Shult) multiplication \( \cdot \) and with adjacency translation \( \tau \) such that the identity matrix \( I \) and the all-one matrix \( J \) belong to \( A \). It is not difficult to show that the partition of \( X \times X \) defined by the equivalence relation

\[ (x, y) \sim (x', y') \iff A_{x} = A_{y} \quad \text{for all } A \in A \]

turns \( X \) into a coherent configuration in the sense of Haman [10]; conversely, the adjacency algebra of a coherent configuration is a coherent algebra [10, 23]. In particular, if \( V \) is a valuation scheme in the sense of Delorme [1], and \( \mathcal{F} \) is its Bose-Mesner algebra (cf. 23, where only the symmetric case is treated). See also Unger [21].

Many properties of coherent algebras and related configurations are formally dual and in some cases, this is due to the existence of a duality operator.

We call a coherent algebra \( V \) self-dual if there is a semi-linear duality operator \( \psi \) on \( V \) such that

\[ \psi(a) = a^T \quad \text{for all } A \in A \]

\[ \psi(AJ) = \psi(A) \quad \text{for all } A \in A \]

\[ \psi(A^T) = (A^T)^T \quad \text{for all } A \in A \]

\[ \psi(ABV) = \psi(A)\psi(B) \quad \text{for all } A, B \in A \]

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Note that then $A = (B^* A B)^T$, so

$p^* = \lambda$, $p^* = \lambda = \lambda$,

$\left(A B^* \right)^T = \frac{1}{2} \left(A B^* + B A^* \right)^T = \frac{1}{2} \left(B A^* + A B^* \right)^T$

is particular $V$ is commutative.

**Example 1.** Let $G$ be a finite Abelian group of order $n$. A decomposition of $G$ into a direct product $G = C \times \ldots \times C$, of cyclic groups $C_i$ of order $n_i$ with generators $g_i$, determines a multiplicative inner product

$\langle x, y \rangle = \prod_i \langle x^i, y^i \rangle$,

where $\langle \cdot, \cdot \rangle$ is a primitive $n_i$-root of unity) with the following properties:

$\langle x, y \rangle = \langle y, x \rangle$,

$\langle x, y \rangle^{-1} = \langle y, x \rangle^{-1}$,

$\langle x, y \rangle = \langle y, x \rangle$,

$\langle x, y \rangle = 1$ for all $x, y \in G$.

Let $G = x = 1$,

$\sum_{x \in G} \langle x, x \rangle = 1$ if $x = 1$,

$\sum_{x \in G} \langle x, x \rangle = 0$ otherwise,

since

$\sum_{x \in G} \langle x, x \rangle = \frac{1}{n} \sum_{x \in G} \langle x, x \rangle = 0$.

In particular, the map $x \mapsto \langle x, \cdot \rangle$ is an isomorphism between $G$ and its character group $\hat{G}$.

The semi-linear map $\alpha$ obtained by extending the definition

$\alpha^* = \sum_{y \in G} \langle x, y \rangle y$ with $\langle \alpha^* x, y \rangle = \sum_{z \in G} \langle x, z \rangle y$

to the group algebra $G \mathbb{F}$ satisfies

$\alpha^* = \sum_{x \in G} (\alpha x)^y = \sum_{x \in G} (\alpha x)^{y^*} = \sum_{x \in G} \langle x, y \rangle y^* = \sum_{x \in G} \langle x, y \rangle (y, y^*) y = \sum_{x \in G} \langle x, y \rangle y = \sum_{x \in G} \langle x, y \rangle (y, y^*) y = \sum_{x \in G} \langle x, y \rangle y$.

Let $V'$ be the co-algebra of $G$ (cf. Wielandt [14]) in its regular action (i.e., the set of all matrices that are invariant under $G$), $V' = \sum_{x \in G} (\alpha x)^y = \sum_{x \in G} \langle x, y \rangle y$, for all $\alpha, \beta \in G$, is isomorphic to the group algebra of $G$ via the isomorphism $\alpha = \sum_{x \in G} \langle x, y \rangle y = \sum_{x \in G} \langle x, y \rangle y = \sum_{x \in G} \langle x, y \rangle y = \sum_{x \in G} \langle x, y \rangle y$, $\alpha \in V'$ is a co-algebra, and the map $\alpha$ defined by $\hat{\alpha} = \hat{\alpha} \circ \alpha$ is a co-linearity operator on $V'$.
Example 2. An association scheme is of Latin square type (negative Latin square type) if every $(0, 1)$-matrix with zero diagonal contained in the Bose-Mesner algebra $\mathcal{M}$ is the adjacency matrix of a pseudo Latin square graph (negative Latin square graph). Here a pseudo Latin square graph is a strongly regular graph with negative $\Delta = \delta - 1$ (cf. Cameron et al. [7]); in particular, imprimitive strongly regular graphs (i.e., disjoint unions of cliques and multipartite graphs) are degenerate pseudo Latin square graphs. E.g., the points of an affine plane (or more generally a net (Bush [6])), together with a partition $(\mathcal{T}, \mathcal{L})$ of the line directions (points at infinity) define an association scheme of Latin square type by calling $x, y \in \mathcal{L}$ associated if $x$ and $y$ are on a line with direction $\mathcal{T}_x$. See also [19, 24].

Define for $A \in \mathcal{M}$ the numbers $k_j, d_j$ by

$$M = k_j I, \quad A \cdot I = d_j I.$$  

Then for any association scheme of Latin square type, the map $v$ with

$$\tilde{v} = \pm d_j \cdot (t \in \mathbb{T})^{-1} (k_j \in \mathbb{N}) (\mathcal{T}_j \in \mathcal{L})$$  

is a valuation (upper sign for Latin square type, lower sign for negative Latin square type) in a duality operator.

Proposition 1. A commutative coherent algebra $F$ has a unique basis $D_1, \ldots, D_r$ of $D$, such that $D_i^\ast = D_i$ for all $i$. $D_i \cdot D_j = \cdots = D_r \cdot D_1 = 0$.

Proof. Every finite dimensional commutative algebra has a basis of idempotents. Let $D_1, \ldots, D_r$ be such a basis with respect to the $\ast$-multiplication. Then each $D_i$ is a $(0)$-idempotent, and since $D_i D_j = 0$ for $i \neq j$, the linear combinations of the $D_i$ representing $I$ must be $\sum_i D_i D_i \cdot D_i D_i$. Similarly, $D_i$ and $I$ are represented by a sum of $D_i$ ($i = 1, \ldots, r$) which is zero in case only one term, and by a suitable permutation we may take $D_1 = I$.

From now on, let $F$ be nilpotent and hence commutative.

Proposition 2. Let $F = (v, \mathcal{T})$, be the matrix defined by

$$D_j = \sum_{v} v_j D_i.$$  

Then $D_j$ has row sum $\kappa_j = \kappa_0$, and we have the relations

$$TT = vI,$$

$$v_j \kappa_0 = v_0 \delta_0,$$

Proof. $(D_j^2) = \sum_{j=0}^\infty v_j^2 D_j^2 = \sum_{j=0}^\infty v_j^2 D_j D_j$ but also $D_j^2 = \kappa_j D_j$, so $TT = vI$. Next we have

$$D_j D_j = \sum_{j=0}^\infty v_j^2 D_j = \kappa_0 D_0 = \kappa_0 I.$$
hence  
\[
D_j = \frac{1}{\langle D_j, D_j \rangle} (D_j D_j)^T = (c_{ij} I) \circ \tau_j, \quad \text{giving } c_{ij} = c_{ji}.
\]
Finally,  
\[
D_j D_j = \frac{1}{\langle D_j, D_j \rangle} (\langle D_j, D_j \rangle I) = \frac{1}{\langle D_j, D_j \rangle} \langle D_j, D_j \rangle I = c_{ij} I
\]
since \(D_j D_j = \mathbb{A}_{ij} \mathbb{B}_{ij}\), hence  
\[
\tau_j D_j = \tau_j (\mathbb{A}_{ij} \mathbb{B}_{ij}) = \left(\mathbb{A}_{ij} \mathbb{B}_{ij}\right)^T = \frac{1}{\langle D_j, D_j \rangle} (D_j D_j)^T
\]
\[
= \frac{1}{\langle D_j, D_j \rangle} (D_j D_j)^T = \tau_j D_j = c_{ij} I.
\]

Since \(D_j D_j = \text{tr} \tau_j D_j = \frac{1}{\langle D_j, D_j \rangle} \langle D_j, D_j \rangle I = \tau_j D_j = c_{ij} I\), and we get \(\tau_j D_j = \tau_j \mathbb{A}_{ij}\). \(\blacksquare\)

**Proposition 3.**

(i) \(A\) has constant row norm so \(k=\mathbb{A}\) has constant diagonal entries.

(ii) \(A\) is a \((0,1)\)-matrix of reducibility \(k=\mathbb{A}\) is isomorphism of rank \(k\).

(iii) \(A\) is nonnegative \(\Rightarrow \mathbb{A}\) is positive semidefinite.

(iv) The \(\frac{1}{\alpha} B_i\) are the irreducible isomorphisms of \(V\).

(v) \(B_i\) has the eigenvalue \(\alpha_i\) with multiplicity \(\delta_i\) \((i=0,\ldots,\alpha)\).

**Proof.**

(i) \(A\) is diagonalizable \(\Rightarrow \mathbb{A}\) is diagonalizable, hence \(\alpha_i=\delta_i=\frac{1}{\alpha_i} \mathbb{A}\), and \(\tau_i D_i = k_i I\), where  
\[
\tau_i = \frac{1}{\alpha_i} \alpha_i D_i = \frac{1}{\alpha_i} \mathbb{A} D_i = \mathbb{A} D_i,
\]
and all \(\alpha_i=0=\mathbb{A}\) and all \(\alpha_i=\mathbb{A}\) is positive semidefinite.

(ii) \(A\) is nonnegative \(\Rightarrow \mathbb{A}\) and all \(\alpha_i=0=\mathbb{A}\) and all \(\mathbb{A}\) is positive semidefinite.

(iii) We have  
\[
\frac{1}{\alpha_i} B_i = \frac{1}{\alpha_i} (D_i D_i)^T = \frac{1}{\alpha_i} D_i D_i.
\]
If \(\frac{1}{\alpha_i} D_i = \frac{1}{\alpha_i} \mathbb{A} + \frac{1}{\alpha_i} P_i\) is a decomposition of \(\frac{1}{\alpha_i} D_i\) into isomorphisms then \(B_i = \mathbb{F}\) is a decomposition into \(\mathbb{F}\), isosystems. But \(B_i\) is a basis and so this is impossible.

(iv) \(B_i = \sum_{j=0}^{\alpha_i} \tau_j D_j\) implies \(\sum_{j=0}^{\alpha_i} \tau_j D_j = B_i\), and since \(\frac{1}{\alpha_i} D_i\) is isomorphism, \(B_i\) has eigenvalues \(\delta_i\) with multiplicity \(\delta_i\) \(\Rightarrow \frac{1}{\alpha_i} D_i = \frac{1}{\alpha_i} \mathbb{A} + \frac{1}{\alpha_i} P_i\) is a decomposition of \(\frac{1}{\alpha_i} D_i\) into isomorphisms.

The eigenvalues of a nonnegative coherent algebra (equivalently, of a symmetric association scheme) are the extremal \(P=(P_j)_{ij}\) and \(Q=(Q_j)_{ij}\), taking the basis of \((0,1)\)-matrices \(D_i\) and the basis of isomorphisms \(B_i\) as follows:  
\[
D_i = \frac{1}{\alpha_i} \mathbb{A}, \quad \tau_i = \frac{1}{\alpha_i} \mathbb{A} D_i.
\]
Of course, \( P=Q=P \), and by the above results, we have \( P=Q=P \) for a self-dual algebra. Conversely, a commutative coherent configuration with \( P=Q \) is self-dual, since the semi-linear map \( \alpha \) with \( \alpha(P)=0 \) is a duality operator.

If \( P \) is a coherent subalgebra of a self-dual coherent algebra, then so is its dual \( P' \); the valuations \( k_i \), multiplications \( f_i \), and eigenoperators \( P' \) of \( P' \) are related to the valuations \( k_i \), multiplications \( f_i \), and eigenoperators \( P \) of \( P \) by

\[
\begin{align*}
  k_i' &= k_i, \quad f_i' = k_i, \quad P' = Q, \quad Q' = P.
\end{align*}
\]

In particular, we have

**Theorem 1.** If a self-dual coherent algebra contains the adjacency matrix of a strongly regular graph \( \Gamma \) with parameters \( k, \lambda, \mu, \rho \) and multiplicity \( 1, f = \lambda - \mu, \) then it also contains the adjacency matrix of a strongly regular graph \( \Gamma' \) with the dual parameters:

\[
\begin{align*}
  \lambda' &= k, \quad \mu' = f, \quad \rho' = \frac{k - \lambda - \mu}{f}, \quad f' = k.
\end{align*}
\]

In particular, the hypothesis can only be satisfied if either \( \Gamma \) is a conference graph, or \( f = \lambda - \mu \) is an integer divisible by 2 (then the known necessary conditions already imply that \( (\lambda - \mu)(\lambda - k) \)).

**Proof.** The eigenvalues of \( \Gamma \) are [cf. Lemma 2.2 of Cameron et al. [7]]

\[
\begin{align*}
  P = & \begin{pmatrix} 1/2 & 1/2 & 1/2 \\ 1/2 & 1/2 & 1/2 \\ 1/2 & 1/2 & 1/2 \end{pmatrix}, \quad Q = \begin{pmatrix} 1/2 & 1/2 & 1/2 \\ 1/2 & 1/2 & 1/2 \\ 1/2 & 1/2 & 1/2 \end{pmatrix}
\end{align*}
\]

where \( k = -1 + k, \lambda = -1 + \lambda, \) and a similar formula holds for the dual. Now it is clear that \( \lambda' = \lambda \) and the relations \( P=Q, \ Q'=P \) give \( k' = f, f' = f, \) and \( \mu' = \frac{k - \lambda - \mu}{f}. \)

Now, using e.g. the formulas in Lemma 2.1 of Cameron et al. [7], we find

\[
\begin{align*}
  \lambda' &= \frac{(\lambda + 1)}{2} - \frac{(k + 1) + (k + 1)}{2} \frac{(r + 1)}{(r - 1)} = \frac{k - \lambda - \mu}{f}, \\
  \mu' &= \frac{(\lambda + 1)}{2} - \frac{(k + 1) + (k + 1)}{2} \frac{(r + 1)}{(r - 1)} = \frac{k - \lambda - \mu}{f}, \\
  f' &= \frac{(\lambda + 1)}{2} - \frac{(k + 1) + (k + 1)}{2} \frac{(r + 1)}{(r - 1)} = f.
\end{align*}
\]

\textbf{Corollary 1 (Delsarte [3])}. If a strongly regular graph \( \Gamma \) has a regular abelian automorphism group then there is a dual strongly regular graph \( \Gamma' \) such that their parameters are related by \( A \). \hfill \Box

\textbf{Corollary 2 (Delsarte [3])}. The Bose-Mesner algebra \( \mathcal{A} \) of a strongly regular graph \( \Gamma \) is self-dual if \( v = (\quad \lambda - \mu \quad) \) if \( \Gamma \) is a conference graph, a pseudo Latin square graph, or a negative Latin square graph. \hfill \Box
Next we review the adjacency matrices of graphs as a self-dual coherent algebra with their duals, which are distance matrices (see Neumaier [12], [13], [14]). We shall also establish a relation with distance regular graphs (see Biggs[4]).

We recall that a distance matrix $A$ is a $n 	imes n$ matrix with nonnegative entries, indexed by a set $X$ with $n$ elements, such that the distance $d(x,y)$ between two elements $x$ and $y$ is $(A_{xy} - 1)$ for all $x, y \in X$. If $A$ has no repeated rows then this turns $A$ into a matrix space, and universally, for every metric $d$ on a finite set $X$, the matrix $D - d(x,y)$ is a distance matrix. A distance matrix $A$ has degree $k$ if $A$ has precisely $k$ distinct nonzero entries, and strength (at least) $k$ if for all $i, j, k$ with $0 \leq i, j, k \leq k$ the $(i,j)$-entry of the product of $C^i_{0}A^j_{0}C^k_{0}$ is a polynomial in $x$ of degree $\min(k, i, k)$ in $x$. We write this as $C^i_{0}A^j_{0}C^k_{0}$. A distance matrix $C$ is spherical if $0=\text{tr}(C^i_{0}A^j_{0}C^k_{0})$ is positive semidefinite for $k=0$. Distance matrices are closely related to spherical $k$-designs defined by Brouwer et al. [4].

Proposition 4. Let $C$ be a distance matrix with $k$ rows.

(i) $C$ has strength 1 if

$$G = \begin{bmatrix} k \end{bmatrix}$$

for some $k > 0$.

(ii) $C$ has strength 2 if $G = \begin{bmatrix} k \end{bmatrix}$ satisfies

$$G^2 = \begin{bmatrix} 0 \end{bmatrix}$$

for some $k > 0$.

(iii) $C$ has strength 3 if (i) or (ii) hold, and

$$G = \begin{bmatrix} k \end{bmatrix}$$

for some $k > 0$.

(iv) $C$ is the distance matrix of a spherical 3-design if $C$ is spherical and (i) holds.

Proof. See Neumaier [13].

Distance matrices with strength 1 for all $i, j$ are called $k$-arcs (see Biggs[4]). A distance regular graph is a graph $F$ such that the distance relation turns $F$ into a distance matrix. If $F$ is a distance regular graph, then the distance matrix $D$ is a distance matrix $A$ such that $A^i_{0}A^j_{0}A^k_{0}$ is a polynomial in $x$ of degree $\min(k, i, k)$ in $x$. We write this as $A^i_{0}A^j_{0}A^k_{0}$. A distance matrix $C$ is spherical if $0=\text{tr}(A^i_{0}A^j_{0}A^k_{0})$ is positive semidefinite for $k=0$. Distance matrices are closely related to spherical $k$-designs defined by Brouwer et al. [4].

Theorem 2. Let $A$ be a symmetric matrix with nonnegative diagonal and constant row sums $k$. Then

(i) $C$ is the distance matrix of strength 1 if $A$ is nonnegative.

(ii) $C$ has strength 2 if $A$ is a $(0,1)$-matrix, i.e. the adjacency matrix of a regular graph of valency $k$.

(iii) $C$ has strength 3 if $F(A)$ is an edge-regular graph, i.e. every edge of $F(A)$ is a constant number of triangles.
(iv) \( C_F \) is the distance matrix of a spherical 3-design iff \( \Gamma(A) \) is a triangle-free graph.
(v) \( C_F \) is a Delaunay matrix iff \( \Gamma(A) \) is distance regular.
(vi) \( C_F \) has repeated rows iff \( \Gamma(A) \) is disconnected.

Proof.

(i) \( A \) is nonnegative \( \implies A^T \) positive semidefinite \( \implies C_F \) is positive semidefinite \( \implies \) \( C_F \) is a spherical 3-design. Conversely, \( C_F \) is a spherical 3-design \( \implies \) \( C_F \) is positive semidefinite \( \implies \) \( (I_k - C_F) \) is positive semidefinite \( \implies \) \( (I_k - C_F)^T \) is positive semidefinite \( \implies \) \( A \) is nonnegative.

(ii) \( C_F \) has strength \( 2k - 2k = A = A^T = \lambda \) (a \((0,1)\)-matrix).

(iii) \( A \) has strength \( 3k(\alpha - \beta)^2 - \alpha + \beta \) \( \implies \) \( \alpha \) is constant \( \implies \) \( A = \Gamma(A) \) is edge regular.

(iv) \( \Gamma(A) \) with constant \( \lambda \).

(v) \( C_F \) is a Delaunay matrix \( \iff \Gamma(A) \) \( \iff \) \( A = \alpha \).

(vi) \[ C_F = \sum \frac{\alpha_x \beta_y}{\beta_y} \frac{1}{2} \frac{(x-y)}{A} \] has eigenvalues \( \lambda, \lambda, \frac{1}{2} \).

Remark 1. Similarly \( \Gamma \) \( \iff \) \( A = \alpha \).

Remark 2. A graph of diameter \( d \) is a Moore graph iff \( \Gamma(A) \) is distance regular.

Remark 3. For the following facts see e.g. Barnes and Sloane [1]:

Let \( \Gamma \) be a graph of diameter \( d \) and degree \( \lambda \), then equality holds if \( \Gamma \) is a Moore graph. (This can be taken as the definition of a Moore graph of degree \( d \).)
and in particular, there is none for $r \geq 5$. A graph $\Gamma$ of diameter $r$ and valency $k$ contains $w = \nu_{r}(k) = \frac{1}{k^{2} - 1} \binom{r}{2}$ points, with equality if $\Gamma$ is a Moore graph, 
\[ \nu_{r}(k) \] is a polynomial of degree $r$ in $k$. Finally, if the valencies of a primitive association scheme are ordered such that $k_1 \geq k_2 \geq \ldots \geq k_r$, then $\nu_r(k_r) = 1$, and equality implies that the graph with valency $k_r$ corresponding to the first relation has girth $r + 1$ with $r^{2} = 4$.

Compare this with properties of a spherical set $X$ of points, discussed in Delsarte, Goethals and Seidel [9]. Suppose that $X$ has $s$ distances and is a spherical $s$-design. Then $r \geq 2s$, and equality holds if $X$ is a tight design. Every tight design is a Delone space; and the tight designs with $r = 2s$ are the polynomials; there are only a few tight designs with $r = 1$, and in particular, there is none for $s \geq 12$. (See Brouwer and Danesi [3], [4].) A set $X$ with $s$ distances in the unit sphere of $\mathbb{R}^r$ contains
\[ w = \nu_s(r) = \frac{r(r - 1)}{s} \binom{r - 1}{s - 1} \]
points, with equality if $r$ is a tight design; $\nu_s(r)$ is a polynomial of degree $s$ in $r$.

Finally, if the multiplication of a primitive association scheme are ordered such that $f_1 \leq f_2 \leq \ldots \leq f_r$, then it can be shown that $f_s = \frac{1}{s} \left( \nu_r(f_s) \right)$, and equality implies that the set of points corresponding to the idempotents of rank $f_r$ is spherical $s$-design.

Some immediate questions arise: How do the analogies work? Why are the formulas slightly different in the two cases? Is there a spherical analogue of $k_1, k_2, \ldots, k_r$ for $s = n$? Is there a Delsarte matrix analogue of the equation $k_1 k_2 \cdots k_r = n!$ obtained by Tutte and Levit [9] for distance regular graphs?

I want to dwell on some remarks on Delsarte's setting for duality. The basic observation is that if $G$ is a $s \times s$ matrix such that $G^r - G = I_s$ and $0 = (0)(G)(I_s)(G)$ is a partition of the columns then the matrices $E_s = G^r - G$ are mutually orthogonal idempotents, and hence define an algebra. Hence we might well call two objects of $s \times s$ multiplication dual if there is a matrix $G$ with $G^r - G = I_s$ of such that suitable partitions of the columns give the first algebra and a suitable partition of the rows gives the second algebra. Finally, we call two association schemes dual (as Delsarte's sense) if their Bose—Mesner algebras are dual with respect to a matrix $G$, and if, in addition, the columns partition is given by the association of a suitable point of the first scheme. Then it can be shown that the partitions of a dual pair of association schemes again satisfy the relations $k_1 f_1 = k_2 f_2$ and $G' = G$.

In the case of a regular regular group, Delsarte's duality is equivalent to our duality, since we may take $G = I_s$, $f = 1$, and the spherical point defining the partition can be taken to be zero.

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