Invitation to Mathematical Elasticity

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This series of lectures will be devoted to mathematical foundations of nonlinear and linear elasticity. We will start with the basic setting of the problem, with the notion of deformation and displacement, and with the axiom of the existence of the stress vector in elastic solids. Further, we are going to discuss Lagrangean description, deformation gradients, measures of deformation and transformation rules for line, surface and volume elements. We will also analyze equilibrium equations in the reference and deformed configurations of the body and discuss various definitions of stress (Cauchy, Piola-Kirchhoff, Kirchhoff). Special attention will be paid to applications of the theory of compensated compactness to nonlinear (hyper)elasticity (weak convergence of minors of deformation gradients) and to various convexity properties of stored energy functions. We will also prove the existence of equilibrium states for polyconvex materials with various additional properties.

We will assume some basic knowledge of Lebesgue and Sobolev spaces as well as of elementary linear functional analysis (Hahn-Banach theorem, weak convergence, Riesz theorem, Lax-Milgram lemma). No physical background is needed but may be advantageous.

Some references

[1] Antman, S.S.: Nonlinear Problems of Elasticity, Springer, New York, 1995.
[2] Ciarlet, P.G.: Mathematical Elasticity. North-Holland, Amsterdam, 1988.
[3] Haupt, P.: Continuum mechanics and theory of materials. Springer, 2000.