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Contents

Additional abstracts of talks

Here are the abstracts we received too late to be included in the Book of Abstracts.

Hidden orbits in discontinuous maps: how periods one and two imply chaos

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(joint work with **Mike Jeffrey**)

Discontinuous maps appear naturally in many areas of nonlinear dynamics. In some situations, they act as approximate models of systems where the rules governing the dynamic behavior undergo a fast (but continuous) change at some borders in the state space, while in other situations, the change of the rules at the boundaries is in fact discontinuous. A novel approach for the investigation of discontinuous maps has been recently suggested in [1]. The key idea of the approach is to extend the definition of a discontinuous map in such a way that at the discontinuities, the function is considered to be *set-valued* (in particular, for 1D maps, *interval-valued*). Note that the orbits of the map remain single-valued: An orbit visiting a discontinuity is mapped to a point belonging to the corresponding set; if an orbit visits the discontinuity again, it may be mapped to the same or to a different point. Accordingly, in addition to all orbits existing in the original discontinuous map, every time an orbit of such an extended map visits a discontinuity, an infinite number of forward orbits (so-called *hidden orbits*) is created. By construction, a *hidden orbit* is an orbit including points inside the discontinuities, and if a hidden orbit $\{x_n \mid n = 1, 2, \dots\}$ satisfies $x_{n+p} = x_n$ for all n, the orbit forms a *hidden cycle* of period p. Clearly, each hidden cycle is repelling. Moreover, to compute a hidden cycle is a simple task, as its points are given by preimages of the corresponding discontinuity.

In the present talk, we focus on the following applications of the proposed approach:

- How hidden orbits simplify bifurcation analysis for discontinuous maps, providing – against expectations – an unified description of bifurcations occurring in continuous and discontinuous maps [1, 2].
- A discontinuous map may act as a model of a system with a very fast but continuous switching process. In such cases, some of the dynamics of the modeled system are lost in the maps, while hidden orbits help to restore these dynamics. [2].
- By definition, a map with vertical branches is discontinuous but connected. Several fundamental theorems have been proven for continuous maps and do not apply to discontinuous ones. However, one may ask whether the requirement for continuity of the function may be relaxed and whether the connectedness would be sufficient as well. This is the case for the Sharkovsky theorem [3].
- Moreover, the existence of a hidden fixed point and a hidden 2-

cycle implies the existence of hidden cycles of all periods, which can be interpreted as an unexpected form of the well-known rule, namely “periods one and two imply chaos” [4].

References

[1] M. Jeffrey and S. Webber. The hidden unstable orbits of maps with gaps. Proc. R. Soc. A, 476(2234):20190473, 2020

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[3] P. Glendinning and M. Jeffrey. Hidden dynamics for piecewise smooth maps. Nonlinearity 34:3184-3198, 2021

[4] M. Jeffrey and V. Avrutin. Periods 1+2 imply chaos in steep or nonsmooth maps. Nonlinearity (submitted 2021)