Model theory of differential-henselian pre-H-fields

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The Ohio State University

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A differential-henselian pre-H-field

Let $\mathbb{T}$ be the differential field of logarithmic-exponential transseries.

- Example series in $\mathbb{T}$:

$$7e^{e^{x}+e^{x/2}+e^{x/4}+\cdots}-3e^{x^2}+5x^{\sqrt{2}}-(\log x)^{\pi}+42+x^{-1}+x^{-2}+\cdots+e^{-x}$$

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Question: What is the model theory of K^*?

Preliminary answers

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- K^* is Liouville closed.
- K^* is differential-henselian rather than newtonian.
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Theorem (PC)

$\mathrm{Th}(K^*, \mathrm{res}(K^*); \pi)$ *is model complete.*

Pre-H-fields

- A **pre-H-field** is an ordered valued differential field K such that:
  1. $\mathcal{O}$ is convex with respect to $\leqslant$;
  2. for all $f \in K$, $f > \mathcal{O} \implies f' > 0$;
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Existentially closed pre-H-fields

Fact (ADH): Every pre-H-field extends to an H-field.

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Theorem (Aschenbrenner–van den Dries–van der Hoeven)

1. *The theory T^{nl} of newtonian, ω-free, Liouville closed H-fields is the model companion of the theory of pre-H-fields.*
2. $T^{\mathrm{nl}}_{\mathrm{small}} = T^{\mathrm{nl}} +$ *"small derivation" axiomatizes the theory of $\mathbb{T}$.*

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Not every pre-H-field with small derivation can be extended to an H-field with small derivation, so $T^{\mathrm{nl}}_{\mathrm{small}}$ is not the model companion of the theory of pre-H-fields with small derivation.

Differential-henselianity

Let K be a pre-H-field.

- K is **differential-henselian** if:
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- If K is differential-henselian, then K has **gap** 0: for all $f \in K^{\times}$, if $f \prec 1$, then $f' \prec 1$ and $f'/f \succ 1$.

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- K is **d-Hensel-Liouville closed** if it is:
  1. differential-henselian;
  2. real closed;
  3. **closed under exponential integration**: for every $a \in K$, there exists $f \in K^\times$ such that $f'/f = a$.

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Theorem (PC)

The theory of d*-Hensel-Liouville closed pre-H-fields with closed ordered differential residue field:*

1. *is the model completion of the theory of pre-H-fields with gap* 0*;*
2. *has quantifier elimination, so is complete and decidable.*

Existentially closed pre-H-fields with gap 0

From quantifier elimination, we get:

Corollary

The theory of d*-Hensel-Liouville closed pre-H-fields with closed ordered differential residue field:*

1. *is distal, and hence has NIP;*
2. *is locally o-minimal.*

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Theorem and corollary do not apply to K^* as above, since $\operatorname{res}(K^*)$ is not a closed ordered differential field.

Ax–Kochen/Ershov theorem for pre-H-fields

A pre-H-field K is **d-Hensel-Liouville closed** if it is:

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Theorem (PC)

Let K_1 and K_2 be d*-Hensel-Liouville closed pre-H-fields with ordered differential residue fields $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$. Then*

$$K_1 \equiv K_2 \iff \boldsymbol{k}_1 \equiv \boldsymbol{k}_2.$$

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Proposition (Hakobyan, 2018)

There exist valued differential fields K_1 and K_2 such that

1. *K_1 and K_2 are monotone and differential-henselian;*
2. *$\boldsymbol{k}_1 \cong \boldsymbol{k}_2$ (as differential fields) and $\Gamma_1 \cong \Gamma_2$ (as ordered groups);*
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Any AKE theorem for valued differential fields with small derivation requires extra assumptions or structure.

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There exist pre-H-fields K_1 and K_2 such that:

1. *K_1 and K_2 are differential-henselian and real closed;*
2. $\boldsymbol{k}_1 \cong \boldsymbol{k}_2$ *(as ordered differential fields);*
3. $(\Gamma_1, \psi_1) \equiv (\Gamma_2, \psi_2)$;
4. $(K_1^\times)^\dagger \neq K_1$ *and* $(K_2^\times)^\dagger = K_2$ *(in particular,* $K_1 \not\equiv K_2$*).*

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Theorem (PC)

For any ordered differential field $\boldsymbol{k}$ that is real closed, closed under exponential integration, and linearly surjective, there exist pre-H-fields $K_1 \subseteq K_2$ such that:

1. *K_1 and K_2 are differential-henselian and real closed;*
2. *$\boldsymbol{k}_1 = \boldsymbol{k}_2 \cong \boldsymbol{k}$ (as ordered differential fields);*
3. *$(\Gamma_1, \psi_1) \preccurlyeq (\Gamma_2, \psi_2)$;*
4. *$(K_1^\times)^\dagger \neq K_1$ and $(K_2^\times)^\dagger = K_2$.*

Two-sorted results

To study structures like K^*, work with two-sorted structures $(K, \boldsymbol{k}; \pi)$ s.t.:

1. K is a d-Hensel-Liouville closed pre-H-field.
2. $\boldsymbol{k}$ is an expansion of the ordered differential residue field of K;
3. $\pi\colon K^2 \to \boldsymbol{k}$ is defined by $\pi(x, y) = \operatorname{res}(xy^{-1})$ if $x \preccurlyeq y \neq 0$ and 0 o.w.

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1. *If* $\operatorname{Th}(\boldsymbol{k})$ *is model complete, then so is* $\operatorname{Th}(K, \boldsymbol{k}; \pi)$*.*
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More precise quantifier reduction

Work with two-sorted structures $(K, \boldsymbol{k}; \pi)$, where:

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Let x be an m-tuple of K-variables and y be a tuple of $\boldsymbol{k}$-variables.

Theorem (PC)

Every formula $\phi(x, y)$ is equivalent to a disjunction of formulas of the form $\theta(x) \wedge \psi(x, y)$, where $\theta(x)$ is a quantifier-free K-formula and $\psi(x, y)$ is special.

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$\psi(x, y)$ is **special** if $\psi(x, y)$ is

$$\psi_{\mathrm{r}}\big(\pi(P_1(x), Q_1(x)), \ldots, \pi(P_k(x), Q_k(x)), y\big),$$

for some $\boldsymbol{k}$-formula $\psi_{\mathrm{r}}(v_1, \ldots, v_k, y)$ and some P_1, Q_1, $\ldots$, P_k, Q_k in $\mathbb{Z}\{X_1, \ldots, X_m\}$.

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Corollary

1. *Every subset of $\boldsymbol{k}^n$ definable in $(K, \boldsymbol{k}; \pi)$ is definable in $\boldsymbol{k}$.*
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Future work

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- Imaginaries in d-Hensel-Liouville closed pre-H-fields.
    - Rideau eliminated imaginaries in existentially closed monotone valued differential fields in the geometric language of Haskell–Hrushovski–Macpherson expanded by a derivation.

Thank you!