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Asymptotic Theory of Transaction Costs

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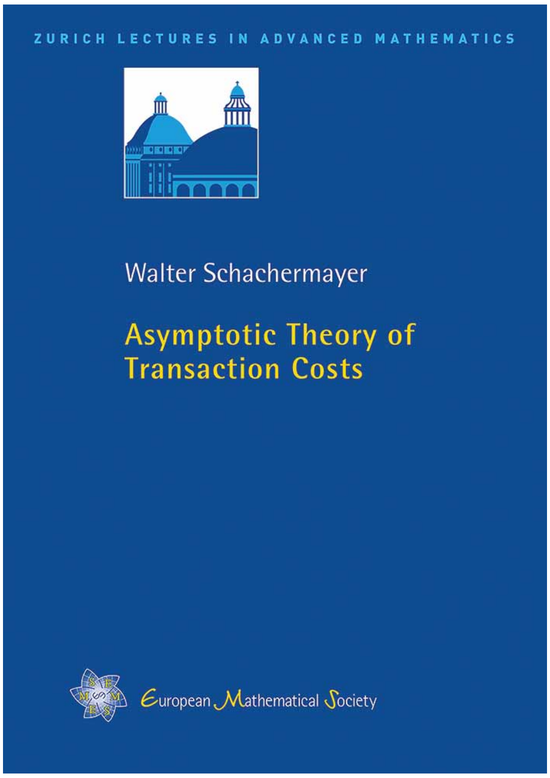
Book review



Asymptotic Theory of Transaction Costs, by Walter Schachermayer, European Mathematical Society (2017). Hardback. ISBN 978-3-03719-173-6.

In *Asymptotic Theory of Transaction Costs*, Professor Walter Schachermayer develops a duality/martingale-based theory of stochastic portfolio optimization subject to fixed proportional transaction costs $\lambda > 0$. The emphasis is on asymptotic behaviour, meaning here that λ is arbitrarily small.

The author introduces duality methods and the notion of shadow price in the easier setting of a finite probability space. The treatment of duality methods extends the discussion of the frictionless case in Delbaen and Schachermayer (2006). The shadow price is a fictitious price process. It equates the solution of a stochastic optimization problem *without* transaction costs subject to the evolution of the shadow price with the solution of a stochastic optimization problem *with* transaction costs subject to the evolution of bid and ask prices. As a result, the shadow price takes its value between the bid and ask prices.

The author then shifts the discussion to a general probability space with an insightful example: logarithmic utility maximization in a Black-Scholes setting, where the price of the risky asset follows a geometric Brownian motion. Logarithmic utility maximization is usually easier to solve than for other utility functions. It also reveals essential connections between primal and dual methods, between the Kelly criterion and general utility maximization, and between pricing and portfolio selection. The author uses this example to derive all the relevant parameters and functions, either directly or as powers of $\lambda^{1/3}$ through a Maclaurin expansion. He also explains the construction of the shadow price process heuristically.

Chapters 4 and 5 are central to the book. The author establishes general and local duality methods in finance. He describes general duality for continuous stochastic processes. Then he shows how traditional assumptions can be advantageously replaced by their local version. A property holds locally if it holds along an unbounded sequence of increasing stopping times. These two chapters are the cornerstone in the construction of a general theory of portfolio optimization subject to proportional transaction costs. General and local duality methods lead to the existence of a shadow price process. Each key result in Chapters 6 and 7 flows clearly from previous developments. The duality Theorem 6.2 and the two-way crossing property (Definition 6.4) combine into the local duality result (Theorem 6.5). Theorem 6.5, the definition of shadow price processes (Definition 7.1), and the property of shadow price process as optimizers (Property 7.2) yield the existence of a shadow price process in Theorem 7.3.

The book culminates in Chapter 8 with a case study of fractional Brownian motions. Theorems 8.4 prove the existence of a shadow price process when the bid and ask prices are modelled using (exponential) fractional Brownian motions. This result is significant because fractional Brownian motions fail to be semimartingales. They allow arbitrage opportunities and rule out standard no-arbitrage pricing arguments. To circumvent this problem, Professor Schachermayer eschews the standard properties of semimartingales and develops an ingenious argument built on the two-way crossing property. The two-way crossing property precludes obvious immediate arbitrage opportunities (Definition 5.10). Combined with a local version of the Fundamental Theorem of Asset Pricing (Theorem 5.11), the two-way crossing property implies the existence of a shadow price process. The mathematical treatment is particularly elegant, linking stochastic analysis, (local) duality theory, optimization, and the geometry of polar and polyhedral sets.

Professor Schachermayer is well known in mathematical finance for his substantial contributions to stochastic analysis, asset pricing theory and portfolio optimization subject to transaction costs. He set a clear objective for the book, that is to provide the necessary background to prove the existence of a shadow price process for fractional Brownian motions. More than this, I found that the book serves as a remarkably clear and insightful introduction to the often-technical literature on the application of duality methods to portfolio optimization with proportional transaction costs.

The derivations in the book are not easy stuff. Most readers, even those well trained in mathematics and quantitative finance, may find some parts of the book challenging. But Professor Schachermayer's pedagogical qualities shine throughout, making a technical subject less daunting and more intuitive. The book undoubtedly benefited from Professor Schachermayer's extensive experience on the subject and from the lectures he gave at the University of Vienna and ETH Zurich.

The book ends with Theorems 8.4, 8.5 and their corollary. I would therefore strongly recommend rereading the preface to change vantage point and capture the bigger picture once more. Those readers who are patient enough to read this book in detail and to work through the results and proofs carefully will find genuine gold.

To conclude, I highly recommend Professor Schachermayer's *Asymptotic Theory of Transaction Costs* to mathematical finance researchers, PhD students, and anyone interested in transaction costs, duality theory or fractional Brownian motions. They will find challenges, inspiration, and a clear guide to the literature.

References

Delbaen, F. and Schachermayer, W., *The Mathematics of Arbitrage*, 2006 (Springer-Verlag: Berlin).

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