Publ. I.R.M.A. Strasbourg, 1987, 340/S–15
Actes 158^e Séminaire Lotharingien, p. 129–130

ON A PROBLEM ABOUT COVERING LINES BY SQUARES

BY

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Abstract. — Let S be the square $[0,n]^2$ of side length $n \in \mathbf{N}$ and let $\mathcal{S} = \{S_1, \ldots, S_t\}$ be a set of unit squares lying inside S, whose sides are parallel to those of S. The set $\mathcal{S}$ is called a line cover, if every line intersecting S also intersetcs some $S_i \in \mathcal{S}$. Let $\tau(n)$ denote the minimum cardinality of a line cover, and let $\tau'(n)$ be defined in the same way, except that we restrict our attention to lines which are parallel to either one of the axes or one of the diagonals of S. It has been conjectured by L.F. Tóth that $\tau(n) = 2n + 0(1)$ and I. Barányi and Z. Füredi that $\tau(n) = \frac{3}{2}n + 0(1)$. We will prove instead, $\tau'(n) = \frac{4}{3} + 0(1)$, and as to Tóth's conjecture, we will exhibit a "non integer" solution to a related LP-relaxation, which has size equal to $\frac{3}{2} + 0(1)$.

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