Some new results on the average shape of binary trees and other simply generated families

This talk continues some of the considerations presented by the author at the meeting at St.Croix-aux-Mines, December 1981.

Let B denote a special family of planted plane trees and B_n the trees of B with exactly n nodes. Considering the leaves of each tree in B_n enumerated from left to right, we ask for the average height a(n,j) of the j-th leaf of a tree in B_n (where all such trees are regarded equally likely): These quantities are of interest in Theoretical Computer Science, as they determine the relative maxima of the height of the stack while performing the simpliest recursive algorithm called "visiting", "traversing" or "exploring" the nodes of a tree in B_n .

In the case of extended binary trees F. Ruskey (1) has given some complicated relations involving a(n,j). We present a relatively simple explicit formula for a(n,j) in this instance.

In the second part we investigate the (finite) limits $a(j)=\lim_{n \to \infty} a(n,j)$ so-called all planted plane trees with labels taken in a fixed set, where for a node of out-degree t there are only a fixed number c_t of possible labels. Our theorem contains a recent result by R. Kemp (2) as a special case.

Literatur:

- (1) F. Ruskey: "On the average shape of binary trees, SIAM J. Algebraic Discr. Methods <u>1</u> (1980), 43-50
- (2) R. Kemp: "On the average oscillation of stack", Combinatorica 2 (1982), 60-66
- (3) P. Kirschenhofer: "On the height of leaves in binary trees", preprint
- (4) P. Kirschenhofer: "On the average shape of simply generated families of trees, J. Graph Theory, to appear 1983
- (5) P. Kirschenhofer, H. Prodinger: "On the average hyperoscillations of planted plane trees, Combinatoria, to appear in issue <u>2</u>
 (2) 1982