A COORDINATIZATION OF GENERALIZED QUADRANGLES OF ORDER (s-1,s+1)

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A generalized quadrangle is an incidence structure $S = (P,B,I)$ with P and B sets of objects called points and lines resp., with a symmetric incidence relation I which satisfies :

(i) each point is incident with $1+t$ lines ($t \geq 1$) and two distinct points are incident with at most one line;

(ii) each line is incident with $1+s$ points ($s \geq 1$) and two distinct lines are incident with at most one point;

(iii) for each point x and each line L, $x \not{I} L$, there exists a unique pair $(y,M) \in P \times B$ such that $x \; I \; M \; I \; y \; I \; L$.

We call (s,t) the order of S.

Let us consider a generalized quadrangle S of order $(s-1,s+1)$ containing a spread R (i.e. a subset R of B such that each point is incident with a unique line of R).

R is called a spread of symmetry for the generalized quadrangle S if the group G_R of automorphisms of S fixing R linewise, acts transitively on each line of R. If S has a spread of symmetry, then from S there arises a generalized quadrangle S' of order s having a center of symmetry. So in this case we are able to give a coordinatization of S, using a planar ternary ring. and G_R, which is derived from the coordinatization of S' due to S.E. Payne [3].

We also investigate the converse problem. Given a planar ternary ring and a group G, which are the conditions to obtain a generalized quadrangle of order $(s-1,s+1)$. Examples are given for the known models $AS(q)$, $T_2^*(O)$ and the dual of $P(T_2(O),x)$ with x a point of the oval O.

Bibliography

[1] M. De Soete and J.A. Thas, A coordinatization of generalized quadrangles of order (s,s+2), to appear in J. Comb. Th. (A).

[2] S.E. Payne, Quadrangles of order (s-1,s+1) J. Algebra 22 (1972), 97-119.

[3] S.E. Payne, Generalized quadrangles of even order, J. Algebra 31 (1974), 367-391.

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