An equivalence of multistatistics on permutations

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PASEP

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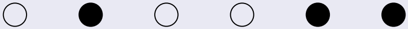

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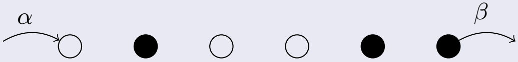

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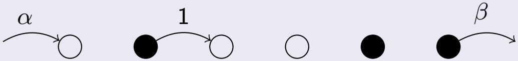

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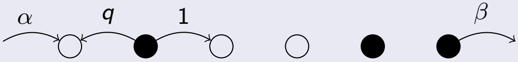

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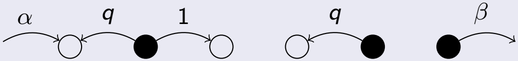

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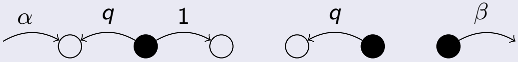

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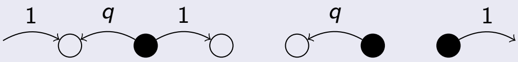

We associate the composition $(2, 3, 1, 1)$ to the above step of the PASEP.

Combinatorial study of the PASEP

The PASEP is closely related with permutations. Let I be a composition associated to a state of the PASEP, the steady-state probability of this state is given by $\sum_{\mathsf{GC}(\sigma)=I} q^{\mathrm{tot}(\sigma)}$ renormalized to make it a probability.

- $\mathsf{GC}(\sigma)$ (*Genocchi composition*) is the descent composition of the values of σ
- $\mathrm{tot}(\sigma)$ is the number of 31-2 patterns in σ.

Tevlin's basis (2007)

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Combinatorial interpretation of Tevlin's basis

GC \ Rec	4	31	22	211	13	121	112	1111
4	1234							
31		1243, 1423 4123	1342 3412		2341	2413		
22			1324 3124		2314			
211			3142	1432, 4132 4312		2431 4231	3241	
13					2134			
121						2143 4213	3421	
112							3214	
1111								4321

Theorem (Hivert, Novelli, Tevlin, Thibon, 2009)

For I a composition of n, we have $R_I = \sum_J G_{IJ} L_J$ where G_{IJ} is equal to the number of permutations σ satisfying $Rec(\sigma) = I$ and $\mathrm{GC}(\sigma) = J$.

q-analog of Tevlin's basis (2010)

Novelli, Thibon, and Williams defined a q-analog of **NCSF** where the transition matrix from $L_I(q)$ to $R_J(q)$ is given by the following matrix:

$$\begin{pmatrix} 1 & . & . & . \\ . & 1+q & 1 & . \\ . & . & 1 & . \\ . & . & . & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & . & . & . & . & . & . & . \\ . & 1+q+q^2 & 1+q & . & 1 & q & . & . \\ . & . & 1+q & . & 1 & . & . & . \\ . & . & q & 1+q+q^2 & . & 1+q & 1 & . \\ . & . & . & . & 1 & . & . & . \\ . & . & . & . & . & 1+q & 1 & . \\ . & . & . & . & . & . & 1 & . \\ . & . & . & . & . & . & . & 1 \end{pmatrix}$$

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Theorem (Novelli, Thibon, Williams, 2010)

For I a composition of n, we have $R_I(q) = \sum_J F_{IJ}(q) L_J(q)$ where:

$$F_{IJ}(q) = \sum_{\substack{\mathrm{Rec}(\sigma)=I \\ \mathsf{LC}(\sigma)=J}} q^{\alpha(\sigma)}$$

Remark

PASEP theory implies that the previous matrix should also be described with the statistics Rec, GC, *and* tot.

Two ways of grouping the permutations

LC \ Rec	4	31	22	211	13	121	112	1111
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Conjecture (Novelli, Thibon, Williams, 2010)

Sending permutations of the left table to $q^{\alpha(\sigma)}$ *gives the same matrix than sending the permutations of the right table to* $q^{\mathrm{tot}(\sigma)}$.

Sketch of proof: let's make some bijections

Involved combinatorial objects

- Permutations;
- Weighted Dyck Paths;
- Subexceedent Functions;
- Decreasing Weighted Subexceedent Functions.

Steps of the bijection

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$$\mathrm{P} \overset{\phi_{FV}}{\longleftrightarrow} \mathrm{WDP} \qquad\qquad \mathrm{P}$$

Catalan

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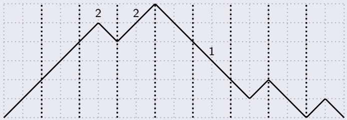
Weighted Dyck paths

A weight for a Dyck path is a word w satisfying for all i, $w_i \leq (h_i - 1)/2$ where h_i is the height of the Dyck path between the $(2i-1)$-th and $2i$-th steps.



The Françon-Viennot bijection: P $\to$ WDP

Let $\sigma \in \mathfrak{S}_n$ we construct $\psi_{FV}(\sigma)$ as follows:

- The $(2k-1)$-th is / iff $k = \sigma_i < \sigma_{i+1}$,
- The $(2k)$-th is / iff $\sigma_{i-1} > \sigma_i = k$.

Moreover, w_k is equal to the number of 31-2 patterns such that k plays the rôle of 2.

Example

$$\phi_{FV}(0.528713649.\infty) =$$

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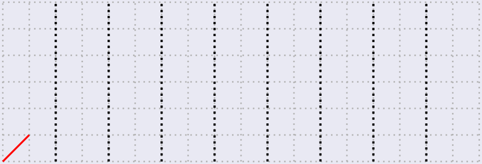
$\phi_{FV}(0.528713649.\infty) =$ 

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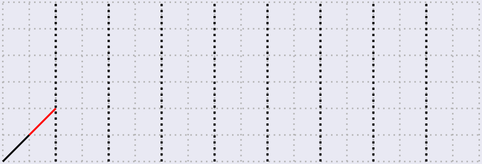
$\phi_{FV}(0.528713649.\infty) =$ 

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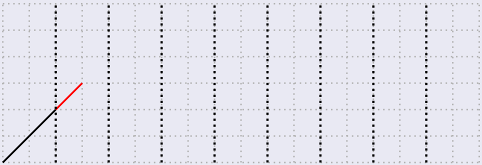

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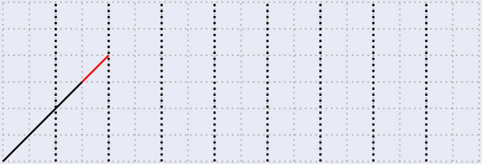
$\phi_{FV}(0.528713649.\infty) =$ 

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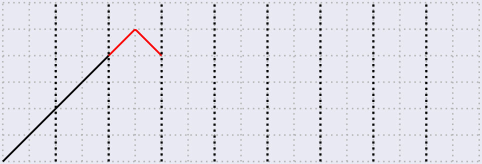
$\phi_{FV}(0.52871364 9.\infty) =$ 

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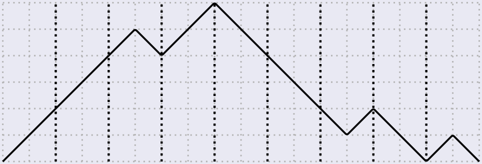

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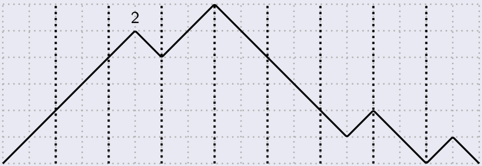

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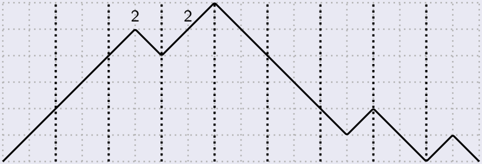

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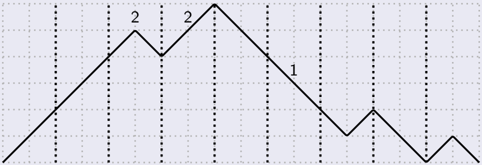

ϕ_1: WDP $\rightarrow$ WDP

ϕ_1 is the involution exchanging  with .

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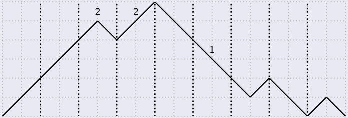

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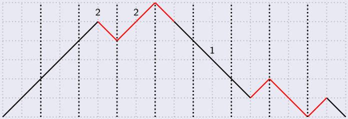

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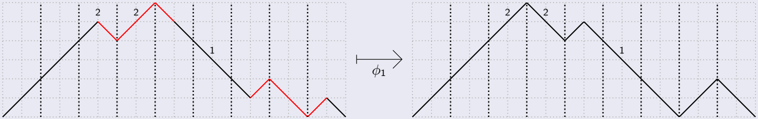

Summary

$$P \overset{\phi_{FV}}{\longleftrightarrow} WDP \overset{\phi_1}{\longleftrightarrow} WDP \overset{\psi_2}{\longleftrightarrow} WDSF \overset{\psi_1}{\longleftrightarrow} SF \overset{Lh}{\longleftrightarrow} P$$

Subexceedent functions

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Bijection with permutations

We use the Lehmer code of the inverse of a permutation σ to construc a subexceedent function f as follows: $f_{\sigma_j} = \#\{i < j | \sigma_i > \sigma_j\}$. For instance,

$$\sigma = 528197634, \ \mathrm{Lh}(\sigma) =$$

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$$\sigma = \overset{\frown}{5}28197634, \ \mathrm{Lh}(\sigma) = 31$$

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Decreasing subexceedent functions

A subexceedent function is decreasing if the word obtained by removing all the zeros is strictly decreasing.
For example, $L = 540300200$.

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- $L = 315503200,\ P = 000000000$

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- $L = 31\mathbf{54}03200$, $P = 000\mathbf{1}00000$, then *pivot* $= 5$;
- $L = \mathbf{5}1\mathbf{2}403200$, $P = 00\mathbf{1}100000$, then *pivot* $= 4$;
- $L = 51\mathbf{41}03200$, $P = 001\mathbf{2}00000$

ψ_1: SF $\rightarrow$ DWSF

- $L = 315503200$, $P = 000000000$, then $pivot = 5$;
- $L = 31\mathbf{54}03200$, $P = 000\mathbf{1}00000$, then $pivot = 5$;
- $L = \mathbf{5}1\mathbf{2}403200$, $P = 00\mathbf{1}100000$, then $pivot = 4$;
- $L = 51\mathbf{41}03200$, $P = 001\mathbf{2}00000$, then $pivot = 4$;

ψ_1: SF $\rightarrow$ DWSF

- $L = 315503200$, $P = 000000000$, then $pivot = 5$;
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- $L = \mathbf{5}1\mathbf{2}403200$, $P = 001\mathbf{1}00000$, then $pivot = 4$;
- $L = 51\mathbf{41}03200$, $P = 001\mathbf{2}00000$, then $pivot = 4$;
- $L = 5\mathbf{40}103200$, $P = 00\mathbf{2}200000$

ψ_1: SF $\rightarrow$ DWSF

- $L = 315503200$, $P = 000000000$, then $pivot = 5$;
- $L = 31\mathbf{54}03200$, $P = 000\mathbf{1}00000$, then $pivot = 5$;
- $L = \mathbf{5}1\mathbf{2}403200$, $P = 001\mathbf{1}00000$, then $pivot = 4$;
- $L = 51\mathbf{41}03200$, $P = 001\mathbf{2}00000$, then $pivot = 4$;
- $L = 5\mathbf{40}103200$, $P = 00\mathbf{2}200000$, then $pivot = 3$;

ψ_1: SF $\rightarrow$ DWSF

- $L = 315503200$, $P = 000000000$, then $pivot = 5$;
- $L = 31\mathbf{54}03200$, $P = 000\mathbf{1}00000$, then $pivot = 5$;
- $L = \mathbf{5}1\mathbf{2}403200$, $P = 00\mathbf{1}100000$, then $pivot = 4$;
- $L = 51\mathbf{41}03200$, $P = 001\mathbf{2}00000$, then $pivot = 4$;
- $L = 5\mathbf{40}103200$, $P = 00\mathbf{2}200000$, then $pivot = 3$;
- $L = 540\mathbf{3}0\mathbf{0}200$, $P = 00220\mathbf{1}000$

ψ_1: SF $\rightarrow$ DWSF

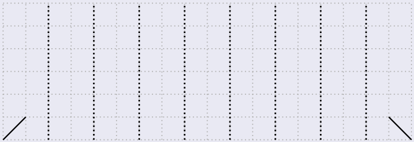
- $L = 315503200$, $P = 000000000$, then $pivot = 5$;
- $L = 31\mathbf{54}03200$, $P = 000\mathbf{1}00000$, then $pivot = 5$;
- $L = \mathbf{5}1\mathbf{2}403200$, $P = 00\mathbf{1}100000$, then $pivot = 4$;
- $L = 51\mathbf{41}03200$, $P = 001\mathbf{2}00000$, then $pivot = 4$;
- $L = 5\mathbf{40}103200$, $P = 00\mathbf{2}200000$, then $pivot = 3$;
- $L = 540\mathbf{3}0\mathbf{0}200$, $P = 00220\mathbf{1}000$ the algorithm stops.

ψ_2: DSF $\rightarrow$ DP

Let $\sigma \in \mathfrak{S}_n$ we construct $\psi_{FV}(\sigma)$ as follows:

- The $(2k)$-th step is $\setminus$ iff $n-k$ is a value of f,
- The $(2k+1)$-th step is $\setminus$ iff $f_k = 0$.

Example

$\psi_2(540300200) =$ 

ψ_2: DSF $\to$ DP

Let $\sigma \in \mathfrak{S}_n$ we construct $\psi_{FV}(\sigma)$ as follows:

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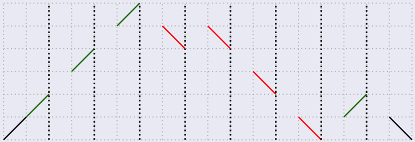

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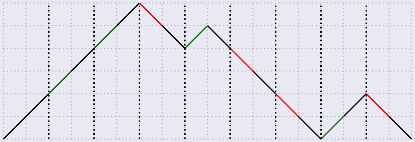

ψ_2: DSF $\to$ DP

Let $\sigma \in \mathfrak{S}_n$ we construct $\psi_{FV}(\sigma)$ as follows:

- The $(2k)$-th step is $\setminus$ iff $n-k$ is a value of f,
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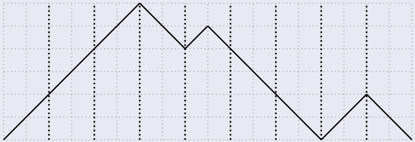
Example

$\psi_2(540300200) =$



Conclusion

Summary

$$\mathrm{P} \overset{\phi_{FV}}{\longleftrightarrow} \mathrm{WDP} \overset{\phi_1}{\longleftrightarrow} \mathrm{WDP} \overset{\psi_2}{\longleftrightarrow} \mathrm{DWSF} \overset{\psi_1}{\longleftrightarrow} \mathrm{SF} \overset{Lh}{\longleftrightarrow} \mathrm{P}$$

Theorem

The map $\phi = Lh^{-1} \circ \psi_1^{-1} \circ \psi_2^{-1} \circ \phi_1 \circ \phi_{FV}$ *is a bijection satisfying*

- $\mathrm{Rec}(\phi(\sigma)) = \mathrm{Rec}(\sigma)$;
- $\mathrm{LC}(\phi(\sigma)) = \mathrm{GC}(\sigma)$;
- $\alpha(\phi(\sigma)) = \mathrm{tot}(\sigma)$.

Perspectives

- Generalisation of the bijection for a larger type of PASEP.
- study of a variant of ϕ_{FV} applied after the involution on weighted Dyck paths implying a third combinatorial interpretation and a new bijection preserving sylvester classes on permutations.

Thank you !