ÜBUNGSAUFGABEN ZU
PROSEMINAR ALGEBRAISCHE TOPOLOGIE

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Exercise 21. Let Σ denote the orientable surface of genus 2, and recall that $\pi_1(\Sigma) \cong \langle a, b, c, d \mid aba^{-1}b^{-1}cdc^{-1}d^{-1} \rangle$. Determine the number of isomorphism classes of two-fold coverings of Σ. How many of these are connected? Hint: Count the equivalence classes of representations of $\pi_1(\Sigma)$ on the set $\{1, 2\}$.

Exercise 22. Let $L = S^{2n-1}/\mathbb{Z}_p$ denote a lense space and recall that $\pi_1(L) \cong \mathbb{Z}_p$. Determine the number of isomorphism classes of (pointed) connected coverings of L. Hint: If $p = p_1^{k_1} p_2^{k_2} \cdots p_l^{k_l}$ is the prime factor decomposition, p_i prime, $k_i \in \mathbb{N}$, then $\mathbb{Z}_p \cong \mathbb{Z}_{p_1^{k_1}} \oplus \mathbb{Z}_{p_2^{k_2}} \oplus \cdots \oplus \mathbb{Z}_{p_l^{k_l}}$ Use this to count the number of subgroups in $\mathbb{Z}_p$.

For further exercises see `http://www.mat.univie.ac.at/~stefan/AT.html`.