32. *Peano's Existence Theorem*
Prove Peano's Theorem (Thm. 3.12) along the lines of the sektch of proof given in 3.15.

33. *Euler-Cauchy Iteration*
Consider the initial value problem

$$x'(t) = t^2 + x(t), \qquad x(0) = 1.$$

By the Euler-Cauchy method calculate an approximation to the solution at time $t = 1$ using a step size of $1/5$ (hence calculate $x_5(1)$ in the notation of Thm. 3.12).

Compare your result with the actual value of the solution at $t = 1$. What is the relative error?

34. *Euler-Cauchy vs. Picard*
Once more consider the initial value problem

$$x'(t) = x, \qquad x(0) = 1.$$

(a) Employ the Euler-Cauchy method to claculate an approximation to the solution at $t = 1$ using step size $1/4$ (hence $x_4(1)$ in the notation of Thm. 3.12).

(b) Compute an approximation of $x(1)$ using Picard's iteration of oder 4 (use problem no. 31).

(c) Compare your results obtained in (a) and (b) with the actual value of the solution at $t = 1$.

35. *Estimates on the time of existence*
Consider the initial value problem

$$x' = x^2, \qquad x(0) = 1.$$

By choosing appropriate values of $\alpha$ and $\beta$ try to make $h$ in the Theorem of Picard and Lindelöf as big as possible. Compare this to the actual maximal interval of existence of the solution.