0.1 Einleitung

In vielen mathematischen Anwendungen spielen partielle Differentialgleichungen (PDEs) eine große Rolle; die Grundgesetze der Physik sind als PDEs formuliert und viele Modellbildungen in den Naturwissenschaften führen auf PDEs. In den meisten Fällen ist diese Formulierung als Differentialgleichung nicht zwingend, sondern von der bequemeren mathematischen Handhabung inspiriert.

Stetige Funktionen—die durchaus sinnvoll physikalische Größen modellieren können—besitzen (im Rahmen der klassischen Analysis) keine Ableitungen; physikalische Zusammenhänge sind oft allgemeiner als ihre Formulierung als PDE. Um diesen bedauerlichen Umstand zu beheben, stehen im Wesentlichen zwei alternative Wege zur Auswahl.

- (Um-)Formulierung der physikalischen Beziehungen als Integralgleichungen (die auch etwa für stetige Funktionen Sinn haben).

- Beibehaltung der PDE-Formulierung und Verallgemeinerung des Ableitungsbegriffs (sodaß auch etwa stetige Funktionen Ableitungen besitzen).

Der zweite Weg führt direkt auf die *Theorie der Distributionen* oder *verallgemeinerten Funktionen*. Neben der Motivation aus den Anwendungen besteht natürlich auch innermathematisches Interesse am „Differenzieren" nichtdifferenzierbarer Funktionen bzw. wurde die Existenz nichtdifferenzierbarer Funktionen als „unangenehm" empfunden. Charles Hermite (1822-1901) schrieb etwa (in einem Brief an Thomas Stieltjes (1856-94)): „I recoil[1] with dismay[2] and horror at this lamentable plague of functions which do not have derivatives".

0.2 Historische Bemerkungen

[aus R.S., „Distributional Methods in General Relativity", 2.1., Dissertation, (2000).]

[...]since it is so tempting, however, we begin with a few historical remarks on the "prehistory" of distribution theory, the details of which may be found in the book of Lützen [7].

Some physicists and also mathematicians were using "generalized functions ideas" thereby for a long time anticipating the later rigorous theory. The names to mention here are most of all J.B. Fourier, G. Kirchhoff and O. Heaviside. P. Dirac in 1926 [2] and later in his famous book [3] introduced the concept of the δ-function which allowed him to draw an analogy between "discrete" and "continuous variables," thereby reaching a unified theory of quantum mechanics combining matrix mechanics and wave mechanics. Since these ideas were so beautiful and convincing the "Dirac-δ" very soon became a widespread tool for physicists. However, the notation δ does not stand for "Dirac" but was originally chosen by Dirac to put emphasis on the analogy between $\delta(p-q)dp$ and the unity matrix which, in physics, commonly is written as the "Kronecker-δ" δ_{pq} (cf. [7], p. 124). On the other hand, descriptions of the δ-distribution as a limit of a series of (smooth) functions go back as far as 1822 and Fourier [5]. Also Kirchhoff [6] already in 1882 fully captured the concept of δ calculating the fundamental solution of the wave operator in $\mathbb{R}^{1+3}$.

A rigorous theory which first (implicitly) used distributions was given in 1932 by S. Bochner [1] while the first definition of distributions in the modern sense (as functionals) appeared in S. Sobolev's 1936 paper [10]. Hence (according to Lützen [7], p. 159ff.) he may be be called the *inventor* of distributions, while finally L. Schwartz *created the theory* of distributions in his classical monograph [9], first published in 1950.

Schwartz' theory rapidly was well received both by mathematicians and physicists who now could use "improper functions" in a well-defined sense. In mathematics, distribution theory was the essential tool to build an elaborate solution concept for linear PDEs. Most notable among the numerous contributions in that field is the famous theorem proved individually by L. Ehrenpreis [4]

[1] implies a start or a movement away through shock, fear or disgust; zurückstoßen

[2] To deprive of courage, resolution and initiative through the preasure of sudden fear or anxiety or great perplexity; Bestürzung

and B. Malgrange [8] guaranteeing the existence of a fundamental solution (in $\mathcal{D}'$) for every non zero constant coefficient linear partial differential operator. [...]

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