Erratum: Semiclassical Asymptotics for the Maxwell-Dirac System

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Due to a computational mistake the polarization conditions (3.18), (3.19), for $u_{0,\pm}$ (or equivalently $u_{0,\pm 1}$) are incorrect. Indeed the correct relations are obtained by choosing $\phi = \phi_-, \text{ i.e. } \phi$ satisfies

$$\partial_t \phi = \sqrt{|\nabla \phi|^2 + 1}.$$ 

Then equation (3.18) is correct (and formally the same but with this new choice of $\phi$) whereas (3.19) has to be replaced by

$$\langle \Pi_+ (-\nabla \phi) u_{0,-1}, (t,x) \rangle = 0 \iff \langle \Pi_- (-\nabla \phi) u_{0,-1}, (t,x) \rangle = u_{0,-1}(t,x),$$

This is then consistent with the corresponding negative phase factor $\exp(-i\phi/\varepsilon)$ used throughout the paper, i.e. instead of (3.20) we get

$$u_0(t,x,\phi(t,x)/\varepsilon) := u_{0,+1}e^{i\phi/\varepsilon} + u_{0,-1}e^{-i\phi/\varepsilon}, \quad u_{0,\pm 1} = \Pi_\pm (\pm \nabla \phi) u_{0,\pm 1}.$$ 

Also we get that $\omega_- (\nabla \phi)$ has to be replaced everywhere by $\omega_- (-\nabla \phi)$ (which indeed happens to be equal $\omega_+ (\nabla \phi)$). The only point where this flaw causes problems is where we have used the orthogonality of $u_{0,-}$ and $u_{0,+}$, which clearly is no longer valid. This implies that on the r.h.s. of (4.1) we get two additional mixed terms of the form

$$\langle u_{0,\pm}(t,x), u_{0,\mp}(t,x) \rangle e^{\mp 2i\phi(t,x)/\varepsilon}.$$ 

However these terms can then be easily dealt with by using the same argument as given in lemma 4.1 for the Zitterbewegung $Z_k$. The electric-potential $V^\varepsilon$ then also becomes an asymptotic expansion similar to the one given for the magnetic potential $A^\varepsilon$ in (4.20) and (4.21). With these modifications all other results remain valid.

We also want to correct two cumbersome typos: In (4.12) the factor $\pm$ should be canceled whereas an additional factor 1/4 has to be included (similarly on the r.h.s. of (4.18)). Also the second expression in (2.18) is not as it should be, but rather has to be replaced by $\Pi_4 - \Pi_- (\xi) = \Pi_+ (\xi)$.

Finally we want to add one sentence of explanation the reader might consider helpful. In Remark 5.3, after (3.17): This clearly implies that we have to assume either $u_{0,-}(0,x)$ or $u_{0,+}(0,x)$ to be identically zero, in order to proceed with an analogous one-phase ansatz (a fact which has already been noted earlier).

References


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