On the Normalization of Combinatorial Proofs for Classical and Intuitionistic Logic

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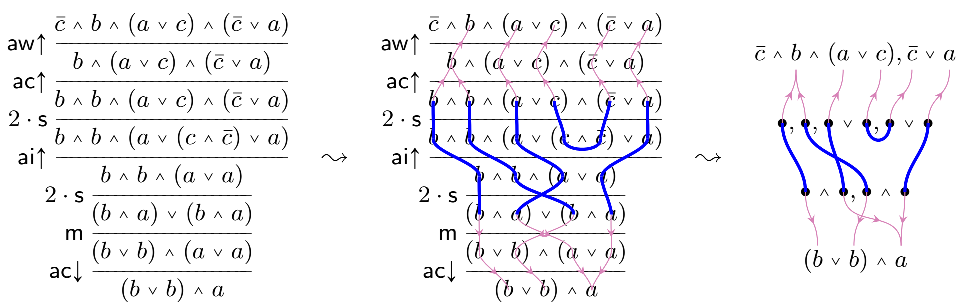
Proof theory is a central area of theoretical computer science, as it can provide the foundations not only for logic programming and functional programming, but also for the formal verification of software. Yet, despite the crucial role played by formal proofs, we have no proper notion of proof identity telling us when two proofs are "the same". This is very different from other areas of mathematics, like group theory, where two groups are "the same" if they are isomorphic, or topology, where two spaces are "the same" if they are homeomorphic.

The problem is that proofs are usually presented by syntactic means, and depending on the chosen syntactic formalism, "the same" proof can look very different. In fact, one can say that at the current state of art, *proof theory is not a theory of proofs but a theory of proof systems*. This means that the first step must be to find ways to describe proofs independent from the proof systems. In other words, we need a "syntax-free" presentation of proofs.

Combinatorial proofs [Hug06a] form such a canonical proof presentation that (1) comes with a polynomial correctness criterion, (2) is independent of the syntax of proof formalisms (like sequent calculi, tableaux systems, resolution, Frege systems, or deep inference systems), and (3) can handle cut and substitution, and their elimination [Hug06b, Str17b]. Below is an example showing how a combinatorial proof can be extracted from a deep inference derivation [Str17a]:



In a nutshell, a combinatorial proof consists of a purely *linear* part (depicted above in blue/bold) and a part that corresponds to *contraction* and *weakening* (depicted above in purple/regular). Combinatorial proofs can be composed horizontally and vertically, and can be substituted into each other.

In this presentation, I will discuss the basic definition of combinatorial proofs, show the differences between the classical and intuitionistic variants, and then discuss various normalization methods.

References

[Hug06a] Dominic Hughes. Proofs Without Syntax. *Annals of Mathematics*, 164(3):1065–1076, 2006.

[Hug06b] Dominic Hughes. Towards Hilbert's 24^{th} problem: Combinatorial proof invariants: (preliminary version). *Electr. Notes Theor. Comput. Sci.*, 165:37–63, 2006.

[Str17a] Lutz Straßburger. Combinatorial Flows and Proof Compression. Research Report RR-9048, Inria Saclay, 2017.

[Str17b] Lutz Straßburger. Combinatorial flows and their normalisation. In Dale Miller, editor, *2nd International Conference on Formal Structures for Computation and Deduction, FSCD 2017, September 3-9, 2017, Oxford, UK*, volume 84 of *LIPIcs*, pages 31:1–31:17. Schloss Dagstuhl - Leibniz-Zentrum fuer Informatik, 2017.