Hybrid Linear Logic, revisited*

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HyLL (Hybrid Linear Logic) is an extension of linear logic (LL) that has been used as a framework for specifying systems that exhibit certain modalities. In HyLL, truth judgments are labelled by *worlds* (having a monoidal structure) and hybrid connectives (`at` and $\downarrow$) relate worlds with formulas. We shall show that HyLL can be deeply encoded in LL. This shows that the use of worlds in HyLL does not increase the expressiveness of LL. Another extension of LL that has extensively been used for specifying systems with modalities is Subexponential Linear Logic (SELL). In SELL, the linear logic exponentials (!, ?) are decorated with labels representing *locations*, and a preorder on such labels defines the provability relation. We propose an encoding of HyLL into SELL$^{\Cap}$ (SELL plus quantification over locations) that gives better insights about the meaning of worlds in HyLL. More precisely, we identify worlds with locations, and show that a flat subexponential structure is sufficient for representing any world structure in HyLL. Finally, we propose the notion of fixed points in multiplicative additive HyLL (μHyMALL), which can be deeply encoded into multiplicative additive linear logic with fixed points (μMALL). As an application, we propose encodings of Computational Tree Logic (CTL) into both μHyMALL and μMALL. In the former, worlds represent states of the transition system, thus exhibiting a pleasant similarity with the semantics of CTL. In the later, states are represented as atoms in the linear context, hence reflecting a more *operational* view of CTL connectives.

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